The Handbook of Mathematics, Physics and Astronomy Data is provided

KEELE UNIVERSITY

EXAMINATIONS, 2011/12

Level III

Friday 13th January 2012, 13.00-15.00

PHYSICS/ASTROPHYSICS

PHY-30028

PHYSICS OF GALAXIES

Candidates should attempt to answer THREE questions.

A sheet of useful formulae and definitions can be found on page 8.

NOT TO BE REMOVED FROM THE EXAMINATION HALL

1. (a) Outline the structure of the Milky Way Galaxy, giving information about the spatial distributions and approximate total masses of the main stellar components, and explaining the basic evidence for dark matter. [40]

(b) At radius R in the midplane of an axisymmetric disc galaxy, where the circular speed is $V_c(R)$, the Oort constants are

$$A \equiv -\frac{1}{2}\left[\frac{dV_c}{dR} - \frac{V_c}{R}\right] \qquad \text{and} \qquad B \equiv -\frac{1}{2}\left[\frac{dV_c}{dR} + \frac{V_c}{R}\right]$$

while the epicyclic frequency is

$$\kappa^2 = \frac{1}{R}\frac{d}{dR}\left(V_c^2\right) + 2\frac{V_c^2}{R^2} .$$

i. What is the epicyclic frequency physically, in the context of nearly circular orbits in the discs of galaxies? [10]

ii. If $\Omega(R)$ is the *angular* speed of a circular orbit of radius R, show that

$$\kappa^2/\Omega^2 = -4B/(A-B) .$$ [25]

iii. What values can B have at radii in a disc where circular orbits are stable? [5]

(c) At the position of the Sun in the Milky Way, the Oort constants have the values

$$A = 14.8 \text{ km s}^{-1}\text{ kpc}^{-1}$$
$$B = -12.4 \text{ km s}^{-1}\text{ kpc}^{-1}$$

Use these numbers, and the information in part (b), to answer the following:

i. Is the Galactic rotation curve locally flat, declining, or rising? [10]

ii. Are nearly circular orbits in the Solar Neighbourhood closed or not? [10]

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2. A model for spherical galaxies with finite total mass $\mathcal{M}$ has, for gravitational potential and the density profile,

$$\Phi(r) \;=\; \frac{G\,\mathcal{M}}{a}\,\ln\left(\frac{r}{r+a}\right) \qquad \text{and} \qquad \rho(r) \;=\; \frac{\mathcal{M}\,a}{4\pi}\,\frac{1}{r^2\,(r+a)^2}\;,$$

where a is a constant with units of radius.

(a) i. Starting from the expression given for $\Phi(r)$ in this model, derive the expression for $\rho(r)$. [20]

ii. Infer an expression for the enclosed mass, $M(r)$, and thus show that a is the half-mass radius in this model. [10]

iii. Show that the circular speed at radius r is given by

$$V_c^2(r) \;=\; \frac{G\,\mathcal{M}}{r+a}\;. \qquad [10]$$

iv. Sketch the circular speed as a function of radius. By considering the behaviour of V_c at small r, give a reason why this model might be suitable for real galaxies. Also, give a physical justification for the behaviour of V_c in the limit of large r. [20]

(b) The total potential energy of a galaxy in this model is

$$W = -\,\frac{G\,\mathcal{M}^2}{2\,a}\;.$$

Write down the general equation you would use to derive this result, and explain the equation in physical terms. (No calculation is required.) [20]

(c) The galaxy NGC 3379 has a total luminosity $L_{\rm tot} = 2\times 10^{10}\ L_\odot$, a measured half-mass radius $r_h = 2.9$ kpc, and an average, one-dimensional internal velocity dispersion $\sigma = 180$ km s^{-1}. Calculate the virial mass of NGC 3379, within the context of the model above. What conclusions can be drawn about dark matter in this galaxy? [20]

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3. The radial temperature dependence $T(r)$ of an accretion disc aro a black hole in an Active Galactic Nucleus is given by

$$T(r) \;=\; \left(\frac{GM\dot{M}}{8\pi\sigma_{\rm SB}\, r^3} \right)^{1/4}$$

where M is the black hole mass, $\dot{M}$ is the mass accretion rate, and $\sigma_{\rm SB}$ is the Stefan-Boltzmann constant.

(a) Assuming that the disc radiates as a blackbody, write down an expression for $F(r)$, the flux over all frequencies, as a function of radius in the disc. [10]

(b) If $R_{\rm in}$ is the innermost radius of the disc, then the total disc luminosity is obtained as

$$L \;=\; \int_{R_{\rm in}}^{\infty} 2 \times 2\pi\, r\, F(r)\, dr \quad .$$

i. Explain this equation in physical terms. [10]

ii. Show that

$$L \;=\; \frac{GM\dot{M}}{2\,R_{\rm in}} \quad .$$ [20]

iii. Explain what the right-hand side of this result corresponds to. [15]

(c) i. Using the expression in part (b)(ii) for the total luminosity, calculate the maximum efficiency, η, of converting rest mass into energy for an accretion disc around a Schwarzschild black hole. [25]

ii. Thus, calculate the maximum luminosity of a quasar with a mass accretion rate of $\dot{M} = 5\ M_\odot\ {\rm yr}^{-1}$. [10]

(d) Under what circumstances might the efficiency η be higher than the value calculated in part (c)? [10]

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4. (a) In a stellar-dynamical version of hydrostatic equilibrium, density profile $n(r)$ and velocity-dispersion profile $\sigma(r)$ of galaxies in a spherical cluster are related to the potential $\Phi(r)$ by

$$\frac{d}{dr}\left(n\sigma^2\right) + n(r)\,\frac{d\Phi}{dr} \;=\; 0 \;.$$

Show that the mass enclosed within radius r follows as

$$M(r) \;=\; -\;\frac{r\,\sigma^2(r)}{G}\left[\frac{r}{n(r)}\,\frac{dn}{dr} + \frac{r}{\sigma^2(r)}\,\frac{d\sigma^2}{dr}\right] \;. \qquad [20]$$

(b) The Coma Cluster contains galaxies with a combined luminosity of $5 \times 10^{12}\ L_\odot$ distributed throughout a spherical volume of radius 2.5 Mpc, with $n(r) \propto r^{-2.3}$ and a constant $\sigma = 1100\ \mathrm{km\ s^{-1}}$.

i. Estimate the mass of Coma within $r = 2.5$ Mpc. [15]

ii. What component of matter likely dominates the mass of the cluster? Explain your reasoning. [15]

(c) The intergalactic gas in clusters of galaxies emits Bremsstrahlung radiation at X-ray wavelengths. The Bremsstrahlung emissivity of a plasma with mean nuclear charge Z, electron number density n_e, and temperature T is $j = 1.4\times10^{-40}\ Z^2\, n_\mathrm{e}^2\, T^{0.5}\ \ \mathrm{W\ m^{-3}}$. Thus, show that the Bremsstrahlung cooling time is

$$t_\mathrm{cool} \simeq 4.7 \times 10^9\ Z^{-2}\ n_\mathrm{e}^{-1}\ T^{0.5}\ \ \text{years}\ . \qquad [20]$$

(d) The intergalactic medium in Coma consists of ionised H and He, for which $Z = 1.2$ and the mean mass per gas particle is $0.6\,m_\mathrm{p}$. The electron number density in this gas follows the relation

$$n_\mathrm{e}(r) = 100\ r_{_\mathrm{Mpc}}^{-2.3}\ \mathrm{m^{-3}} \quad ,$$

where $r_{_\mathrm{Mpc}}$ is the radius from the cluster centre, in units of Mpc.

i. Calculate the virial temperature of the gas in Coma, given that $\sigma = 1100\ \mathrm{km\ s^{-1}}$ for the galaxies in the cluster. [15]

ii. Estimate the cooling radius in Coma. [15]

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5. The diagram given on page 7 illustrates the orientation-dependent unified model of Active Galactic Nuclei (AGN).

   (a) In the optical spectrum of a Type I AGN, broad hydrogen Hβ and narrow oxygen [O III] emission lines are observed. In the spectrum of a Type II AGN, the Hβ and [O III] lines are both narrow.

      i. What types of atomic transition do the [O III] and Hβ lines correspond to, in general? [10]

      ii. Explain the physical conditions under which these lines are produced. Thus, from what region(s) in the diagram on page 7 do you expect the lines to originate in an AGN? [20]

      iii. With reference to the unification diagram, explain why the Hβ line is broad in the Type I AGN but narrow in the Type II AGN, whereas the [O III] line is narrow in both. [20]

   (b) Some Type II AGN are referred to as being "Thomson-thick."

      i. What is meant by this term? [5]

      ii. NGC 1068, a Type II AGN, has a hydrogen column density of $N_H = 1 \times 10^{29}$ m^{-2} along the line of sight to its nucleus. Evaluate the Thomson depth along this line of sight. [10]

      iii. Thus, by what factor is the nuclear emission from NGC 1068 attenuated? Where in the diagram on page 7 would you expect the majority of this attenuation to occur? [10]

   (c) NGC 1068 emits an infra-red luminosity of 1×10^{37} W. If dust sublimates at a temperature of $T = 1500$ K, and assuming that the dusty torus in NGC 1068 covers a total solid angle of 2π steradians, then estimate the inner radius of the torus. What other assumption(s) have you made in your calculation? [25]

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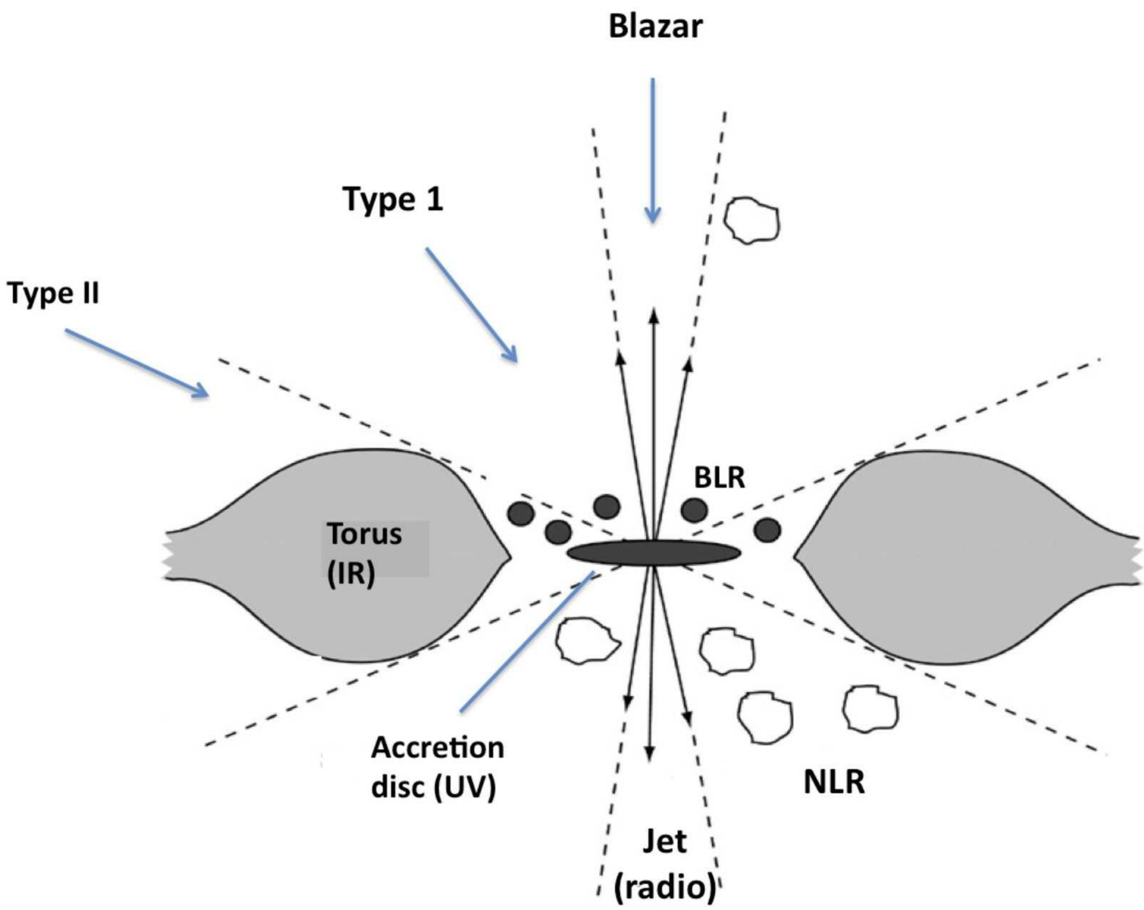
<u>**Figure to be used in answering Question 5**</u>



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Useful formulae and definitions

In ***spherical polar coordinates:***

$$\boldsymbol{\nabla}\Phi \;=\; \hat{\boldsymbol{r}}\,\frac{\partial\Phi}{\partial r} \;+\; \hat{\boldsymbol{\theta}}\,\frac{1}{r}\frac{\partial\Phi}{\partial\theta} \;+\; \hat{\boldsymbol{\phi}}\,\frac{1}{r\,\sin\theta}\frac{\partial\Phi}{\partial\phi}$$

$$\nabla^2\Phi \;=\; \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Phi}{\partial r}\right) \;+\; \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\,\frac{\partial\Phi}{\partial\theta}\right) \;+\; \frac{1}{r^2\sin^2\theta}\frac{\partial^2\Phi}{\partial\phi^2}$$

In ***cylindrical polar coordinates:***

$$\boldsymbol{\nabla}\Phi \;=\; \hat{\boldsymbol{R}}\,\frac{\partial\Phi}{\partial R} \;+\; \hat{\boldsymbol{\theta}}\,\frac{1}{R}\frac{\partial\Phi}{\partial\theta} \;+\; \hat{\boldsymbol{z}}\,\frac{\partial\Phi}{\partial z}$$

$$\nabla^2\Phi \;=\; \frac{1}{R}\frac{\partial}{\partial R}\left(R\,\frac{\partial\Phi}{\partial R}\right) \;+\; \frac{1}{R^2}\frac{\partial^2\Phi}{\partial\theta^2} \;+\; \frac{\partial^2\Phi}{\partial z^2}$$

Stefan-Boltzmann law: $L \;=\; A \times \sigma_{\rm SB}\,T^4$

Schwarzschild radius: $R_{\rm S} \;=\; \dfrac{2GM}{c^2}$

Hubble time: $t_0 \;=\; 1.3\times10^{10}$ years