The Handbook of Mathematics, Physics and Astronomy Data is provided

KEELE UNIVERSITY

EXAMINATIONS, 2011/12

Level III

Wednesday 25th April 2012, 9.30-11.30

PHYSICS/ASTROPHYSICS

PHY-30025

LIFE IN THE UNIVERSE

Candidates should attempt to answer THREE questions.

NOT TO BE REMOVED FROM THE EXAMINATION HALL

1. (a) Give a version of the *Drake equation*, briefly explaining meaning of all of its terms. [20]

   (b) Describe the current constraints on the different terms in your version of the Drake equation, and the evidence on which these are based. [50]

   (c) Describe an experiment – deemed feasible within the next few decades – which could help constrain one of the least constrained terms in your version of the Drake equation. [30]

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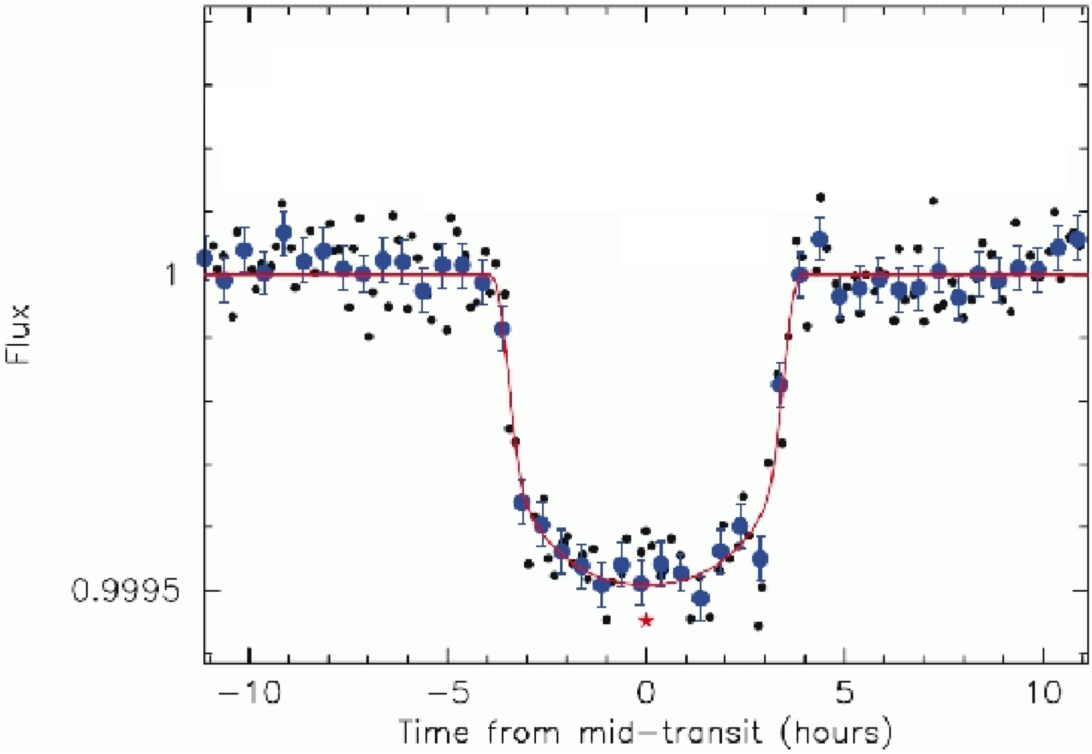
2. Kepler 22 is a star very similar to the Sun. Brightness measureme
of Kepler 22 are presented in the figure below.



(a) Use the figure to determine the size of planet Kepler 22b in Earth radii. [20]

(b) Use the figure to determine the distance between the planet Kepler 22b and its host star Kepler 22 in astronomical units. [30]

(c) Making a reasonable assumption about the possible mass of Kepler 22b, calculate the amplitude of the line-of-sight velocity variations of Kepler 22. [30]

(d) Sketch a fully-labelled diagram of the line-of-sight velocity of Kepler 22 as a function of time for a full orbit of Kepler 22b. Indicate the time of transit. [20]

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3. (a) Describe briefly the origin of ozone in Earth's stratosphere, and why it is considered a biosignature. [20]

(b) Sketch and annotate the spectrum of Earth seen from space, between wavelengths $\lambda = 300$ nm and 20 μm. [20]

(c) Show that the intensity of sunlight penetrating the atmosphere to a depth z is diminished in the following way:

$$I_\lambda = I_\lambda(0)\exp(-\kappa_\lambda z),$$

where κ_λ is the absorption coefficient per unit length. [10]

(d) The optical depth $\tau_\lambda = \kappa_\lambda z$ of the entire atmosphere can also be expressed as $\tau_\lambda = \alpha_\lambda \Sigma$, where α_λ is the absorption cross-section and Σ is the column density. Show that for Earth's atmosphere, $\Sigma \approx 10^4$ kg m^{-2}. [20]

(e) If Earth's atmosphere were liquid, its density would be similar to that of water. Hence show that the depth of this liquid layer would be ≈ 10 m. [10]

(f) The equivalent layer of ozone would only be 3 μm deep. Hence calculate the level of attenuation of sunlight at $\lambda = 250$ nm and $\lambda = 350$ nm if $\alpha_{250\mathrm{nm}} = 10^4$ m^2 kg^{-1} and $\alpha_{350\mathrm{nm}} = 100$ m^2 kg^{-1}. [20]

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4. Consider a disc in Keplerian rotation around the young Sun. disc has inner and outer radii of $R_{\rm in} = 0.1$ AU and $R_{\rm out} = 100$ AU, respectively, a total mass $M = 0.024\ \mathrm{M}_\odot$ and a surface mass density $\Sigma = \Sigma_0 r^{-1.5}$ where r is the distance to the Sun.

(a) Describe briefly how the collapse of a rotating cloud core leads to a proto-planetary disc. [20]

(b) Show that approximately

$$\Sigma_0 \approx \frac{M}{4\pi} R_{\rm out}^{-0.5}. \qquad [20]$$

(c) Show that the angular momentum in the disc is approximately

$$L \approx \frac{M}{2} \left(G M_\odot R_{\rm out}\right)^{0.5}. \qquad [30]$$

(d) Consider a planet within the disc, with $M_{\rm planet} = 10^{-3}\ \mathrm{M}_\odot$, on a nearly circular orbit around the Sun with a radius $r = 5$ AU.

i. Calculate the mass contained within the annulus that is carved out of the disc by the planet's gravitational influence sphere as the planet orbits the Sun, and show that it is less than the mass of the planet. [20]

ii. In light of the answer to part (d)i., describe a mechanism by which the planet could have acquired its mass. [10]

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5. Consider a circular metal sheet with a radius of 1000 km, a surface mass density of 100 kg m^{-2}, and a bond albedo of 0.5. It is placed in orbit around the Sun at a distance of 150 million km, facing the Sun.

   (a) Estimate the effective gravity at the rim of the sheet, and comment on the implications for any humans living on it. [30]

   (b) Calculate the power at which solar radiation is absorbed by the sheet. [10]

   (c) Calculate the equilibrium temperature of the sheet, explaining any assumptions you make. [20]

   (d) Calculate the force on the sheet as a result of the radiation pressure due to the Sun. [20]

   (e) Give an estimate of the amount of rocket fuel needed to maintain its orbit, and compare the energy needed for this with your answer to part (b). [20]