The Handbook of Mathematics, Physics and Astronomy Data is provided

KEELE UNIVERSITY

EXAMINATIONS, 2011/12

Level III

Monday 23rd April 2012, 13.00-15.00

PHYSICS/ASTROPHYSICS

PHY-30003

THE PHYSICS OF COMPACT OBJECTS

Candidates should attempt to answer THREE questions.

NOT TO BE REMOVED FROM THE EXAMINATION HALL

1. (a) Explain what is meant by electron degeneracy and discuss how this leads to the phenomenon of *degeneracy pressure*. [20]

   (b) Sketch a graph of radius versus mass for white dwarf stars made entirely of carbon. Label the axes with numerical values in terms of solar masses and radii and indicate the *Chandrasekhar mass*. [20]

   (c) Using your plot, and assuming that white dwarf stars have internal temperatures of $\simeq 10^7\,\mathrm{K}$, show that the electrons in the interior of a *typical* white dwarf are almost completely degenerate. [25]

   (d) At what density in the gas does the electron degeneracy become relativistic? [15]

   (e) Explain as fully as you can why, for a stable white dwarf, the electrons near the core must be more relativistic than those nearer the surface. [20]

[You may assume that the Fermi momentum is given by $p_F = (3h^3 n/8\pi)^{1/3}$, where n is the fermion number density.]

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2. (a) Using the virial theorem, and explaining any other assumptions you make, show that the radius of a neutron star supported by ideal, non-relativistic degeneracy pressure is approximately

$$R \simeq 12.6 \left(\frac{M}{M_\odot}\right)^{-1/3} \text{ km}$$

[40]

(b) Using your knowledge of the properties of the strong nuclear force, estimate the limiting mass of a neutron star for which the ideal gas assumption is unlikely to be appropriate. [20]

(c) Give an example of how the ideal equation of state for degenerate neutrons might be (i) hardened, or (ii) softened, at high densities. In each case describe how this would affect the radius for a given mass. [20]

(d) Discuss quantitatively whether General Relativity will be an important factor in determining the structure of a neutron star. [10]

(e) The radius of the event horizon for a non-rotating black hole is $2GM/c^2$. Estimate the maximum mass of a neutron star that can have a surface outside the event horizon. [10]

[The pressure of a degenerate gas of non-relativistic fermions is

$$P = \frac{h^2}{20m}\left(\frac{3}{\pi}\right)^{1/3} n^{5/3},$$

where m and n are the fermion mass and number density respectively.]

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3. The average thermal energy *per neutron* inside a neutron sta[r] internal temperature T (in Kelvin) is given approximately by

$$E_{\rm th} = 10^{-3} \left(\frac{T}{10^{10}}\right) \frac{3k_BT}{2}\,.$$

(a) A typical neutron star has $T \leq 10^{10}\,$K. By sketching the neutron occupation index as a function of energy, explain why the thermal heat energy per neutron is much less than the classical value of $3k_BT/2$ but non-zero. [20]

(b) Energy losses from a neutron star of mass M (in kg) are dominated by neutrinos emitted by the modified URCA process, with a luminosity of

$$L_\nu = 3 \times 10^9\, M \left(\frac{T}{10^{10}}\right)^8 \quad {\rm W}.$$

i. What is the modified URCA process? [10]

ii. Write down a differential equation describing the rate of change of internal temperature for a neutron star. Hence show that a neutron star takes approximately 4×10^5 years to cool from $T = 10^{10}\,$K to $T = 10^8\,$K. [40]

(c) About 10^{11} stars have been born in the Galaxy with masses $0.3 \leq M/M_\odot \leq 100$ and with an initial distribution of stellar mass described by $N(M) \propto M^{-5/2}$. Assuming a uniform star formation rate, use your your answer to part (b) to estimate how many Galactic neutron stars there are now with $T > 10^8\,$K. [30]

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4. (a) Describe what is meant by *inverse β-decay* and explain why a *relativistically degenerate* gas of electrons is required for it to occur in white dwarf stars. [30]

(b) Carbon may undergo inverse β-decay at electron threshold energies of 13.9 MeV. At what mass density would this occur in a white dwarf made entirely of carbon? [25]

(c) Using a polytropic equation of state of the form $P \propto \rho^{\alpha}$, it can be shown that the relationship between pressure and energy density, u, is given by

$$P = u(\alpha - 1) \ .$$

Explain what is meant by an equation of state and use the virial theorem to demonstrate that a gravitationally bound star must have $\alpha > 4/3$. [25]

(d) Describe how the occurrence of inverse β-decay modifies the equation of state at high densities; why does this lead to a lower maximum mass for white dwarfs than if they were governed by the ideal equation of state for a degenerate electron gas? [20]

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5. (a) Define what is meant by the Fermi energy E_F, and explain w $E_F = du/dn$ in a degenerate gas with energy density u and particle number density n. [15]

(b) Write down an expression for the total energy density of a gas containing a mixture of degenerate neutrons, electrons and protons and hence show that at equilibrium

$$E_{F,n} = E_{F,p} + E_{F,e}\,,$$

where $E_{F,i}$ is the Fermi energy of species i. [25]

(c) Briefly describe the steps required to calculate the equation of state for the gas in part (b). [15]

(d) At very high densities in a neutron star the following reaction may become possible

$$n \rightarrow p + \pi^- \ .$$

Estimate the threshold neutron number density at which this occurs, clearly stating any assumptions that you make. [25]

(e) Discuss the consequences of pion production in a neutron star core and state how these might affect the observable properties of neutron stars. [20]

[You may assume that the Fermi momentum is given by $p_F = (3h^3n/8\pi)^{1/3}$, where n is the fermion number density. The neutron, proton and pion rest mass energies are given by $m_nc^2 = 939.57\,\mathrm{MeV}$, $m_pc^2 = 938.27\,\mathrm{MeV}$, $m_\pi c^2 = 139.57\,\mathrm{MeV}$.]