The Handbook of Mathematics, Physics and Astronomy Data is provided

KEELE UNIVERSITY

EXAMINATIONS, 2011/12

Level III

Friday, 4th May 2012, 09:30–11:30

PHYSICS/ASTROPHYSICS

PHY-30001

COSMOLOGY

Candidates should attempt to answer THREE questions.

NOT TO BE REMOVED FROM THE EXAMINATION HALL

1. Given the Friedman equation

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2}$$

show that in the radiation era the universe expanded at a rate of $a \propto t^{1/2}$. [20]

Find relations (in the radiation era) between (a) the Hubble parameter and the time since the Big Bang, and (b) the temperature and the time (you may take the mass-energy density of radiation as $\rho_{\rm rad} = \frac{4\sigma}{c^3}T^4$). [25]

Hence estimate the temperature and the energy density at the end of the radiation era, 70 000 years after the Big Bang. [10]

What was the number density of photons at that time? [10]

List (giving approximate abundance ratios relative to photons) the other main constituents of the universe at that time. [20]

Given that the photon temperature has since cooled to 3K, estimate the number density of photons in the universe today. [15]

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2. If some "stuff" has the equation of state $P = w\epsilon$ (where pressure and ϵ is the energy density) then its density scales wit scale factor as

$$\rho = \rho_0 a^{-3(1+w)}.$$

What are the values of w for matter, for radiation, and for the cosmological constant? [15]

Given the Friedman equation,

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \sum_n \rho_w - \frac{kc^2}{a^2}$$

show by substitution that for a universe dominated by one type of such stuff (where $w \neq -1$)

$$a \propto t^q \quad \text{where} \quad q = \frac{2}{3(w+1)}.$$

[20]

Given that, in such a universe, $\rho_w = \rho_c \Omega$ where $\rho_c = 3H^2/(8\pi G)$, show that

$$\Omega - 1 = \frac{kc^2}{a^2 H^2}.$$

[20]

Hence show that in such a universe

$$\Omega - 1 \propto t^{\frac{6w+2}{3w+3}}.$$

[25]

Comment on the implications of this result for normal matter. If w were to be close to -1, what would this imply? Discuss the relevance of this to the "flatness" problem of cosmology. [20]

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3. For a bound, matter-dominated universe the Friedman equa
be written

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2}.$$

Show that the maximum a occurs when it equals α where $\alpha = 8\pi G\rho_0/3kc^2$. [20]

Re-write the Friedman equation using α to simplify it. Hence show that the time taken for the universe to expand to α is given by

$$t_{\max} = \frac{1}{c\sqrt{k}} \int_0^{\alpha} \frac{a^{1/2}}{(\alpha - a)^{1/2}}\, da.$$

[25]

Hence show that the time taken for the universe to reach maximum expansion is

$$t_{\max} = \frac{\alpha\pi}{2c\sqrt{k}}.$$

[20]

Sketch a against time for the above universe. [10]

What is the physical interpretation of the parameter k in the above equations? [10]

Suggest one possible observation that would tell us that we were in such a universe. [15]

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4. Give reasons why Type Ia supernovae are the prime dista… cators used in cosmology.

   Consider a plot of the scale factor, a, against the time, t, for our universe since the Big Bang. Explain why the x-axis can equivalently be plotted as the observed brightness of supernovae, and the y-axis as the redshift z of the supernovae. [20]

   Given the deceleration parameter $q = \frac{\Omega_{\text{matter}}}{2} - \Omega_\Lambda$ and given that $\Omega_{\text{matter},0} = 0.3$ and $\Omega_{\Lambda,0} = 0.7$, sketch the above plot for our universe. State the direction of curvature of the a versus t curve that you expect for (a) relatively recent supernovae, (b) the most distant supernovae. [25]

   By considering how the densities, ρ_{matter} and ρ_Λ, scale with changes in the scale factor, a, estimate the value of the scale factor at the epoch when the universe was neither decelerating nor accelerating (take $a_0 = 1$). [35]

5. (i) Explain why the night sky is dark, despite the standard cosmological model extending to infinity in all directions. [25]

   (ii) Explain how the "Cosmological Principle" leads to Hubble's law as the only acceptable form of expansion. [25]

   (iii) Show how Newtonian gravity leads to the concept of a "critical density" given by $\rho_c = \dfrac{3H^2}{8\pi G}$. [25]

   (iv) Explain the emergence of the Cosmic Microwave Background photons at a specific time as the universe cooled. [25]