The Handbook of Mathematics, Physics and Astronomy Data is provided

KEELE UNIVERSITY

EXAMINATIONS, 2011/12

Level II

Wednesday 11$^{\text{th}}$ January 2012, 09:30 - 11:30

PHYSICS/ASTROPHYSICS

PHY-20027

Optics and Thermodynamics

Candidates should attempt ALL of PART A and ONE question from each of PARTS B and C.

PART A yields 40% of the marks, PART B yields 30%, PART C yields 30%

NOT TO BE REMOVED FROM THE EXAMINATION HALL

PART A Answer all TEN questions

A1 Sketch a Michelson Interferometer, identifying all relevant components. [4]

A2 A Michelson Interferometer is used to produce an interference pattern using a monochromatic light source. Describe the arrangement of the mirrors that produces:

(a) circular fringes; [2]

(b) straight fringes. [2]

A3 The near point of an eye is located 1 m away. Determine the focal length of the lens required to allow clear vision of an object placed 30 cm in front of the eye. [4]

A4 Light from a monochromatic light source is incident on a slit with width $a = 0.75\,\text{mm}$. A diffraction pattern is observed on a screen located at a distance $L = 2\,\text{m}$. The first minimum is measured at a distance $y = 1.35\,\text{mm}$ from the central maximum. What is the wavelength of the light? [4]

A5 State Fourier's theorem. For a generic function $f(x)$ write down its representation as a Fourier series. [4]

A6 Sketch the Carnot cycle in a $P - V$ diagram, stating each of the thermodynamical processes involved. [4]

A7 Explain the meaning of the terms “heat capacity” and “latent heat” and describe the similarity and distinction between them. [4]

A8 State the First Law of Thermodynamics, and use it to show that the free expansion of an ideal gas is an isothermal process. [4]

A9 Consider one mole of molten chocolate at a temperature $T_1 = 24°\text{C}$, warming up in the mouth to body temperature, $T_2 = 36°\text{C}$. The entropy of the chocolate increases by an amount

$$\Delta S_{\text{chocolate}} = C_{\text{V}} \ln \frac{T_2}{T_1},$$

while the change in entropy of the body is

$$\Delta S_{\text{body}} = C_{\text{V}} \left(\frac{T_1}{T_2} - 1 \right).$$

Show that this process is irreversible. [4]

A10 With the help of a suitable diagram, explain the physical meaning of the Clausius–Clapeyron equation. [4]

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PART B Answer ONE out of TWO questions

B1 (a) Show that the object-image relationship for a thin lens of refractive index n_g immersed in water (refractive index n_w) is

$$\frac{1}{s_o} + \frac{1}{s_i} = \left(\frac{n_g}{n_w} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right).$$

where s_o and s_i are respectively the object and image distances, and R_1 and R_2 are the radii of the lens surfaces. [14]

(b) Use the lensmaker's equation to show that the focal length of a glass lens (n_g=1.5) immersed in water (n_w=1.33) is about 4 times the focal length of the same lens in vacuum. [10]

(c) The objective of an astronomical telescope has a 60 cm focal length. The distance between the lenses is 65 cm. What is the angular magnification of this telescope, when the instrument is adjusted for a relaxed eye? [6]

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B2 (a) Consider a rectangular glass block (with refractive index n_g = 1.5) in vacuum. A beam of natural light strikes the first surface at an angle of incidence equal to the polarisation angle. Show that the transmitted beam also strikes the second surface at the polarisation angle. [12]

(b) What is relative orientation of the polarisation axes of two stacked polaroid filters, if the intensity of the incident unpolarised light is reduced by a factor 4? [8]

(c) i. Explain briefly what is meant by the terms linear polarisation and circular polarisation. [5]

ii. Derive the state of polarisation of the wave described by:

$$\mathbf{E} = E_0 \, \cos\left(\frac{\pi}{2} - (kz - \omega t)\right) \, \hat{\mathbf{i}} + E_0 \, \sin(kz - \omega t) \, \hat{\mathbf{j}}$$

[5]

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PART C Answer ONE out of TWO questions

C1 Consider an engine, running on 1 mole of an ideal gas, based on the following cycle:

1. Isothermal compression at temperature T_1, from volume V_{a} to volume V_{b};
2. Ignition, releasing heat Q_2, causing an increase in pressure whilst not being allowed to expand;
3. Isothermal expansion at temperature T_2;
4. Cooling at constant volume to temperature T_1.

(a) Draw this cycle in $P - V$ and $S - T$ diagrams (where P is pressure and S is entropy), and label the diagrams to indicate the four steps. [10]

(b) The efficiency of this engine is given by:

$$\eta = \frac{R(T_2 - T_1)\ln r}{C_{\mathrm{V}}(T_2 - T_1) + RT_2 \ln r},$$

where $r = V_{\mathrm{a}}/V_{\mathrm{b}}$ is the compression ratio and C_{V} is the heat capacity for the exploding gas.

i. By considering the heat absorbed by the gas and the work done by it, derive the above equation for the efficiency of the engine. [15]

ii. Discuss how the efficiency of this engine can be optimised, and compare it with the efficiency of the Carnot cycle. [5]

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C2 The Clausius–Clapeyron equation for the phase transition betw liquid and vapour can be used to find that the boiling point at pressure P has a temperature

$$T_{\text{boil}} = \left(\frac{nR}{L} \ln \frac{P_0}{P} + \frac{1}{T_0} \right)^{-1} ,$$

where T_0 is the boiling temperature at pressure P_0.

(a) Consider 1 kg of water ($\equiv$ 55.4 mole) at 0°C, on a mountain summit where the pressure is half that at sea level. For water, $L = 2.27 \times 10^6$ J kg^{-1} and $C_{\text{V,liquid}} = 4.187 \times 10^3$ J kg^{-1} K^{-1}.

i. Show that $T_{\text{boil}} = 354.4$ K. [3]

ii. Calculate the total energy required to turn the water into vapour. [5]

(b) The water vapour, at its mountain summit boiling temperature, is then taken in a thermally and pressure isolated container to sea level. There it is maintained in thermal isolation but allowed to come into pressure equilibrium with its surroundings. For water vapour, the ratio of specific heats $\gamma = 1.32$.

i. Show that the temperature increases to $T = 419.2$ K. [6]

ii. Calculate the amount of work done on the vapour. [6]

(c) Finally, the vapour is allowed to cool to a temperature of 0°C. For water vapour, $C_{\text{P}} = 1.996 \times 10^3$ J kg^{-1} K^{-1}.

i. Calculate the total energy released by the water. [5]

ii. Compare your answers to parts (bii) and (di), and give a physical explanation for the difference. [5]