

**The Handbook of Mathematics, Physics and
Astronomy Data is provided**

KEELE UNIVERSITY

EXAMINATIONS, 2011/12

Level II

Friday 18th May 2012, 13.00-15.00

PHYSICS/ASTROPHYSICS

PHY-20026

STATISTICAL MECHANICS AND SOLID STATE PHYSICS

Candidates should attempt to answer FOUR questions.

NOT TO BE REMOVED FROM THE EXAMINATION HALL

1. (a) State what is meant by the following terms:

- i. lattice vectors [5]
- ii. unit cell [5]
- iii. Miller indices [5]

(b) A certain lattice is described by the following vectors:

$$\mathbf{a} = 2a \mathbf{i}$$

$$\mathbf{b} = a(\mathbf{i} + \mathbf{j})$$

$$\mathbf{c} = 2a \mathbf{k}$$

where \mathbf{i} , \mathbf{j} , \mathbf{k} are cartesian unit vectors and a is a constant.

- i. Find the volume of the unit cell. [15]
- ii. Find the reciprocal lattice vectors \mathbf{a}^* , \mathbf{b}^* , \mathbf{c}^* . [30]
- iii. Find the volume of the unit cell in the reciprocal lattice. [15]
- iv. Determine the angle between the [111] and [002] planes. [25]

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2. (a) Explain what is meant by a Schottky defect.
 (b) A crystal contains N atoms. Explain why there are

$$\Omega = \frac{N!}{n!(N-n)!}$$

ways of placing n Schottky defects in the crystal. [20]

- (c) A crystal initially contains 10^{20} atoms. Calculate the entropy change ΔS caused by forming 10^{16} Schottky defects in the crystal. [25]

- (d) To form each Schottky defect requires energy E_S , and the formation of n defects results in a change of free energy

$$\Delta F = \Delta E - T \Delta S ,$$

where ΔE is the energy required to form the defects and T is the temperature. Assuming that the most favourable value for n is that which makes ΔF a minimum, obtain an expression for the most favourable value of n in terms of N , E_S and T . [30]

- (e) The energy needed to form a Schottky defect in copper is 0.89 eV. Calculate the number of defects in 1 kg-mole of copper at 800 K. [15]

[N.B. You may assume Stirling's approximation for large n :
 $\ln n! \simeq n \ln n - n$.]

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3. (a) Describe the *classical* theory of the specific heat of solids, and show that, for this case, the molar specific heat $C_V = 3R$, where R is the gas constant. [10]
- (b) State one success of the theory. In what way does the theory fail? [10]
- (c) Describe (without giving mathematical details) the Debye theory of the specific heats of solids. [20]
- (d) The Debye theory gives the following expression for the molar specific heat of a solid at temperature T :

$$C_V = \frac{9RT^3}{\Theta_D^3} \int_0^{\Theta_D/T} \frac{x^4 e^x}{(e^x - 1)^2} dx .$$

- i. Define the quantity Θ_D in this expression; how is it related to a property of the lattice? [5]
- ii. Show that, in the limit $T \gg \Theta_D$, the classical value for C_V is recovered. [20]
- iii. What is the form of C_V for $T \ll \Theta_D$? [20]
- iv. Calculate the molar specific heat of carbon at 10 K, given that $\Theta_D = 2230$ K for carbon. [15]

[You may assume that

$$\int_0^\infty \frac{x^4 e^x}{(e^x - 1)^2} dx = \frac{4\pi^4}{15}]$$

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4. (a) What is meant by the phrase *density of states*? [1]
- (b) Write down an expression for the single particle partition function if the density of states is

$$g(\epsilon) d\epsilon = \frac{4\pi mV}{h^3} [2m\epsilon]^{1/2} d\epsilon$$

and where the symbols have their usual meanings. [10]

- (c) Assuming that the density of states is given by $g(\epsilon) d\epsilon$ in part (b), show that the single particle partition function is

$$Z_{\text{sp}} = V \left(\frac{2\pi m k_B T}{h^2} \right)^{3/2}. \quad [30]$$

- (d) Classical Physics is valid for a system of N particles occupying a volume V provided that $Z_{\text{sp}} \gg N$. Discuss whether Classical Physics adequately describes the following systems:

- i. the gas consisting of protons in the core of a star composed of pure hydrogen; the density and temperature in the core are $2 \times 10^6 \text{ kg m}^{-3}$ and $1 \times 10^8 \text{ K}$ respectively. [25]
- ii. the corresponding electron gas in the core of the same star. [25]

[You may assume that

$$\int_0^\infty x^2 e^{-x^2} dx = \frac{\sqrt{\pi}}{4}]$$

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5. (a) The concentrations of conduction electrons (n) and holes (p) in a semiconductor at temperature T are given by

$$n \simeq N_c \exp[-(E_c - E_F)/k_B T]$$

$$p \simeq N_v \exp[-(E_F - E_v)/k_B T]$$

respectively. State what is meant by the terms N_c , N_v , E_c , E_v , E_F in these expressions. [15]

- (b) The figure on page 7 below shows the energy band for a semiconductor, in the region of the band gap.
- Would you consider that the semiconductor is an n -type or a p -type? Explain your answer. [5]
 - Calculate the probability, at 270 K, that an electron occupies an energy state at the bottom of the conduction band. [15]
 - Calculate the probability, at 270 K, that there is a vacancy in an energy state at the top of the valence band. [15]
 - For this semiconductor, the effective masses are $m_e^* = 0.10 m_e$ and $m_h^* = 0.05 m_e$. Calculate the density of states at the top of the valence band and at the bottom of the conduction band at 270 K. [20]
 - Hence calculate the concentration of holes at the top of the valence band and of electrons at the bottom of the conduction band. [20]
 - Comment on whether this is consistent with your answer to part (b) i. [10]

[N.B. You may assume that the probability of occupancy of a state with energy E is

$$f(\epsilon) = \frac{1}{1 + \exp\left[\frac{E - E_F}{k_B T}\right]}$$

and that

$$N_c = N_v = 2 \left(\frac{m^* k_B T}{2\pi \hbar^2} \right)^{3/2} \quad \text{where } m^* \text{ is the effective mass. } \quad \left. \right]$$

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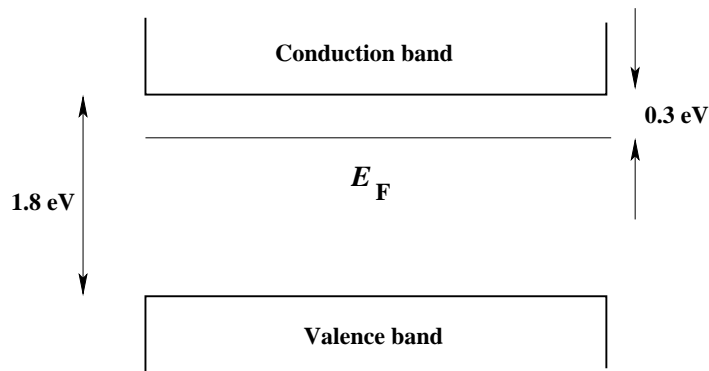


Figure for question 5.

6. (a) Describe briefly what is meant by *ferromagnetism* and *antiferromagnetism*. [10]

(b) i. Describe how the Weiss model postulates the existence of an “internal field” in a ferromagnetic material. Assuming that the magnetisation M at temperature T is given in this case by

$$M = \frac{N\mu^2}{k_B T} B,$$

where B is the *total* field, show that the susceptibility $\chi = M/B_a$, where B_a is the *applied* field, may be written in the form

$$\chi = \frac{C}{T - T_c}$$

where T_c is the critical temperature and C is a constant. [30]

ii. Hence deduce an expression for T_c . [10]

iii. How does the material behave below T_c ? [20]

(c) How is the Weiss model adapted for the case of antiferromagnetism? [30]