EXAMINATION PAPER CONTAINS STUDENT'S A[…]S

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The Handbook of Mathematics, Physics and Astronomy Data is provided

KEELE UNIVERSITY

EXAMINATIONS, 2011/12

Level I

Friday 13th January 2012, 16:00–18:00

PHYSICS/ASTROPHYSICS

PHY-10022

MECHANICS, GRAVITY and RELATIVITY

Candidates should attempt ALL of PARTS A and B, and TWO questions from PART C. PARTS A and B should be answered on the exam paper; PART C should be answered in the examination booklet which should be attached to the exam paper at the end of the exam with a treasury tag. PART A yields 16% of the marks, PART B yields 24%, PART C yields 60%.

Please do not write in the box below

A		C1		Total
B		C2		
		C3		
		C4		

NOT TO BE REMOVED FROM THE EXAMINATION HALL

PART A **Tick one box by the answer you judge to be**

A1 The force acting on a body is equivalent to:

☐ dE/dt ☐ $E \times x$ ☐ m^2v ☐ dp/dt

A2 The units of energy are equivalent to:

☐ $\mathrm{m}^2\ \mathrm{s}^{-2}$ ☐ $\mathrm{kg\ m}^2\ \mathrm{s}^{-2}$ ☐ kg m s ☐ $\mathrm{kg\ m}^2\ \mathrm{s}$ [1]

A3 Given a force we can find the power by multiplying by:

☐ energy ☐ distance ☐ velocity ☐ mass [1]

A4 A particle moving with distance $x = 2t^2$ has a velocity of:

☐ $8t$ ☐ $4t^2$ ☐ $4t$ ☐ 4 [1]

A5 The dot product of the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ is equivalent to:

☐ $ab\cos\theta$ ☐ $\boldsymbol{a} \wedge \boldsymbol{b}$ ☐ $ab\sin\theta\ \hat{\boldsymbol{n}}$ ☐ $\sqrt{a^2+b^2}$ [1]

A6 Conservation of momentum results from the fact that the laws of physics are the same:

☐ at different times ☐ at different speeds
☐ at different angles ☐ in different places [1]

A7 The angular momentum vector points:

☐ along the axis of rotation
☐ perpendicular to the torque
☐ in the plane of the body
☐ in the direction of θ [1]

A8 A flywheel revolves 10 times in 10 seconds. Its angular velocity (in s^{-1}) is:

☐ π ☐ $\pi/2$ ☐ 2π ☐ 1 [1]

/Cont'd

A9 If the angle turned by a wheel is described by $\theta = 30t$ … lar velocity is:

☐ increasing ☐ decreasing ☐ zero ☐ constant

A10 The expression for gravitational potential energy mgh is valid:

☐ always ☐ for small m ☐ for small h ☐ never [1]

A11 Kepler's laws state that planetary orbits:

☐ are always circular ☐ have a fixed speed
☐ don't depend on mass ☐ are elliptical [1]

A12 Two masses of M and $2M$ are separated by r. Midway between the masses the gravitational potential is:

☐ $-12GM/r$ ☐ $-6GM/r$ ☐ $-3GM/r$ ☐ $-3GM/2r$ [1]

A13 Special Relativity applies only to:

☐ heavy bodies ☐ uniform relative motions
☐ accelerating bodies ☐ velocities near c [1]

A14 A spacecraft's proper length is 8 m. An observer sees it approaching at $0.8\,c$. The spacecraft's apparent length is:

☐ 4.4 m ☐ 4.8 m ☐ 5.4 m ☐ 7.2 m [1]

A15 The total energy of a proton (rest mass 1.67×10^{-27} kg) travelling with a γ factor of 2.3 is (in 10^{-10} J):

☐ 7.95 ☐ 3.46 ☐ 1.50 ☐ 0.65 [1]

A16 Newton's laws of motion remain valid in Special Relativity provided the mass is replaced by:

☐ m_0 ☐ γm_0 ☐ mc^2 ☐ βm [1]

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PART B Answer all EIGHT questions

B1 A mass of 10 g travels with a kinetic energy of 2 J. On hitting a target it comes to a halt in 0.1 s. What is the average force exerted by the mass on the target during the collision? [3]

B2 A force of $F = \sqrt{x}$ N, where x is distance, is applied to a particle over the range $0 < x < 5$ m. What is the work done? [3]

B3 If a skier skis down a slope of 55° (to the horizontal); in the absence of friction, what is his acceleration down the slope in terms of g? [3]

B4 Newton's 2nd law can be written $F = dp/dt$ or $F = ma$. Write down statements equivalent to both of these for the quantity torque instead of force. [3]

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B5 A 100-g mass is swung around on a piece of light string
long. What is the moment of inertia?

B6 A meteorite falls from infinity to collide with a planet which has a mass of 10^{25} kg and a radius of 10^{8} m. At what speed does it strike? [3]

B7 What is the momentum of a proton travelling at $0.8c$? [3]

B8 An elementary particle decays after $\tau = 5$ s. It is fired at a speed of 0.999c towards a detector located 10^{10} m away. Will it reach the detector? Justify your answer. [3]

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C1 A particle of mass 0.1 kg has a location as a function of time specified by

$$x = 4t - \frac{1}{2}\cos(2\pi t) \text{ m.}$$

Sketch (with labelled axes) the position, velocity and acceleration of the particle over the range $0 < t < 2$ s. [12]

Find an expression for the force acting on the particle. [3]

What is the power being transferred to the particle at $t = 0.5$ s and at $t = 1$ s? [6]

Evaluate the total work done on the particle over $0 < t < 2$ s. [6]

Evaluate dp/dt at $t = 0.5$ s. [3]

C2 Starting from $I = \int r^2\, dm$ show that a thin rod, length L, mass M, spinning about an axis at one end has a moment of inertia of $I = ML^2/3$. [10]

If such a rod (initially stationary) has a mass of 3 kg, a length of 0.8 m, and is spun up by a torque of 100 N m acting for 10 s, how fast is it now spinning? What is its angular momentum? [6]

A second rod is similar to the first, but has a mass per length, λ, which is zero at the axis of rotation but rises linearly to $\lambda = 6$ kg m^{-1} at the other end. Find I for this second rod. [10]

If this second rod is spun up by the same torque for the same time as the first rod, what is its angular momentum? [4]

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C3 A space station is in a circular orbit 150 km above Earth
Use Newtonian gravity to show that its orbital speed v is gi
$v^2 = GM/r$ (where M is the Earth's mass, of 6.0×10^{24} kg, r is t
orbital radius; Earth's radius is 6380 km). [4]

Find the minimum energy per kilogram needed to launch a payload from Earth to the space station. [8]

A probe is then launched from the space station; find the minimum energy per kilogram needed to send the probe to deep space. [6]

Is there an overall energy advantage or disadvantage in sending a deep-space payload first to the space station and then launching it from there? Justify your answer. [6]

Discuss whether you would need more or less energy to send such a probe to the planet Mercury compared to the planet Jupiter. [6]

C4 Consider a frame S' moving at velocity v along the x-axis with respect to a stationary frame S. Motion u'_x as seen from S' will have an x' coordinate given by $x' = u'_x t'$.

Use this and the Lorentz transforms to show that the same motion viewed from S would have a speed

$$u_x = \frac{v + u'_x}{1 + v u'_x / c^2}.$$

[10]

As seen by one observer, Rocket A travelling at $0.8c$ is being chased by Rocket B travelling at $0.9c$. As viewed from Rocket A, at what relative speed is Rocket B catching him? [10]

Rocket A fires a missile at $0.1c$ (as he sees it) at Rocket B. At what speed does Rocket B see the missile approaching? [6]

Rocket B fires a laser beam at Rocket A; at what speed relative to Rocket A does this beam travel? [4]