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**The Handbook of Mathematics, Physics and Astronomy Data is provided**

KEELE UNIVERSITY

EXAMINATIONS, 2011/12

Level I

Friday 13<sup>th</sup> January 2012, 16:00–18:00

PHYSICS/ASTROPHYSICS

PHY-10022

MECHANICS, GRAVITY and RELATIVITY

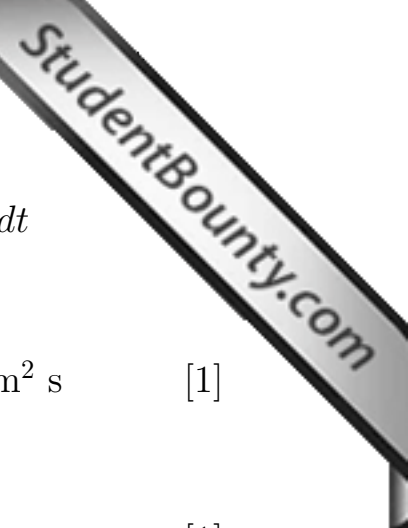
Candidates should attempt ALL of PARTS A and B, and TWO questions from PART C. PARTS A and B should be answered on the exam paper; PART C should be answered in the examination booklet which should be attached to the exam paper at the end of the exam with a treasury tag. PART A yields 16% of the marks, PART B yields 24%, PART C yields 60%.

Please do not write in the box below

A		C1		Total
B		C2		
		C3		
		C4		

**NOT TO BE REMOVED FROM THE EXAMINATION HALL**

**PART A** Tick one box by the answer you judge to be



A1 The force acting on a body is equivalent to:

- $dE/dt$         $E \times x$         $m^2v$         $dp/dt$

A2 The units of energy are equivalent to:

- $m^2 s^{-2}$         $kg m^2 s^{-2}$         $kg m s$         $kg m^2 s$       [1]

A3 Given a force we can find the power by multiplying by:

- energy       distance       velocity       mass      [1]

A4 A particle moving with distance  $x = 2t^2$  has a velocity of:

- $8t$         $4t^2$         $4t$         $4$       [1]

A5 The dot product of the vectors  $\mathbf{a}$  and  $\mathbf{b}$  is equivalent to:

- $ab \cos \theta$         $\mathbf{a} \wedge \mathbf{b}$         $ab \sin \theta \hat{n}$         $\sqrt{a^2 + b^2}$       [1]

A6 Conservation of momentum results from the fact that the laws of physics are the same:

- at different times       at different speeds  
 at different angles       in different places      [1]

A7 The angular momentum vector points:

- along the axis of rotation  
 perpendicular to the torque  
 in the plane of the body  
 in the direction of  $\theta$       [1]

A8 A flywheel revolves 10 times in 10 seconds. Its angular velocity (in  $s^{-1}$ ) is:

- $\pi$         $\pi/2$         $2\pi$         $1$       [1]

A9 If the angle turned by a wheel is described by  $\theta = 30t^2$ , the angular velocity is:

- increasing     decreasing     zero     constant

A10 The expression for gravitational potential energy  $mgh$  is valid:

- always     for small  $m$      for small  $h$      never    [1]

A11 Kepler's laws state that planetary orbits:

- are always circular     have a fixed speed  
 don't depend on mass     are elliptical    [1]

A12 Two masses of  $M$  and  $2M$  are separated by  $r$ . Midway between the masses the gravitational potential is:

- $-12GM/r$       $-6GM/r$       $-3GM/r$       $-3GM/2r$     [1]

A13 Special Relativity applies only to:

- heavy bodies     uniform relative motions  
 accelerating bodies     velocities near  $c$     [1]

A14 A spacecraft's proper length is 8 m. An observer sees it approaching at  $0.8c$ . The spacecraft's apparent length is:

- 4.4 m     4.8 m     5.4 m     7.2 m    [1]

A15 The total energy of a proton (rest mass  $1.67 \times 10^{-27}$  kg) travelling with a  $\gamma$  factor of 2.3 is (in  $10^{-10}$  J):

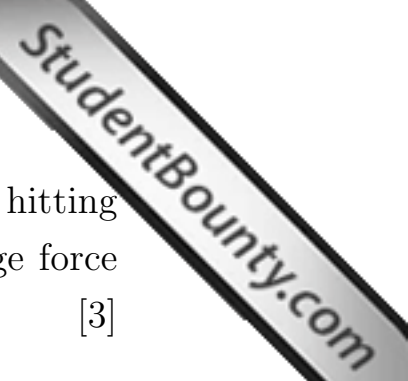
- 7.95     3.46     1.50     0.65    [1]

A16 Newton's laws of motion remain valid in Special Relativity provided the mass is replaced by:

- $m_0$       $\gamma m_0$       $mc^2$       $\beta m$     [1]

/Cont'd

**PART B**      **Answer all EIGHT questions**



B1    A mass of 10 g travels with a kinetic energy of 2 J. On hitting a target it comes to a halt in 0.1 s. What is the average force exerted by the mass on the target during the collision?      [3]

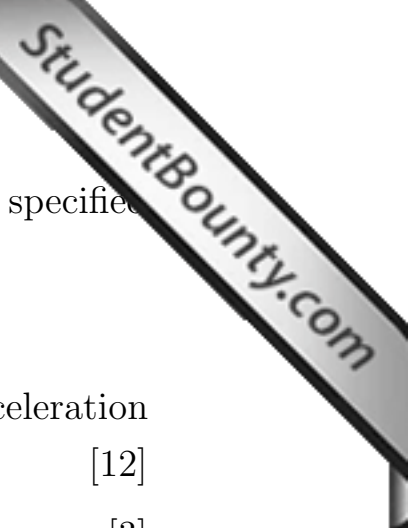
B2    A force of  $F = \sqrt{x}$  N, where  $x$  is distance, is applied to a particle over the range  $0 < x < 5$  m. What is the work done?      [3]

B3    If a skier skis down a slope of  $55^\circ$  (to the horizontal); in the absence of friction, what is his acceleration down the slope in terms of  $g$ ?      [3]

B4    Newton's 2nd law can be written  $F = dp/dt$  or  $F = ma$ . Write down statements equivalent to both of these for the quantity torque instead of force.      [3]

- B5 A 100-g mass is swung around on a piece of light string  $1.0\text{ m}$  long. What is the moment of inertia?
- B6 A meteorite falls from infinity to collide with a planet which has a mass of  $10^{25}\text{ kg}$  and a radius of  $10^8\text{ m}$ . At what speed does it strike? [3]
- B7 What is the momentum of a proton travelling at  $0.8c$ ? [3]
- B8 An elementary particle decays after  $\tau = 5\text{ s}$ . It is fired at a speed of  $0.999c$  towards a detector located  $10^{10}\text{ m}$  away. Will it reach the detector? Justify your answer. [3]

PART C Answer TWO out of FOUR questions



C1 A particle of mass 0.1 kg has a location as a function of time specified by

$$x = 4t - \frac{1}{2} \cos(2\pi t) \text{ m.}$$

Sketch (with labelled axes) the position, velocity and acceleration of the particle over the range  $0 < t < 2$  s. [12]

Find an expression for the force acting on the particle. [3]

What is the power being transferred to the particle at  $t = 0.5$  s and at  $t = 1$  s? [6]

Evaluate the total work done on the particle over  $0 < t < 2$  s. [6]

Evaluate  $dp/dt$  at  $t = 0.5$  s. [3]

C2 Starting from  $I = \int r^2 dm$  show that a thin rod, length  $L$ , mass  $M$ , spinning about an axis at one end has a moment of inertia of  $I = ML^2/3$ . [10]

If such a rod (initially stationary) has a mass of 3 kg, a length of 0.8 m, and is spun up by a torque of 100 N m acting for 10 s, how fast is it now spinning? What is its angular momentum? [6]

A second rod is similar to the first, but has a mass per length,  $\lambda$ , which is zero at the axis of rotation but rises linearly to  $\lambda = 6 \text{ kg m}^{-1}$  at the other end. Find  $I$  for this second rod. [10]

If this second rod is spun up by the same torque for the same time as the first rod, what is its angular momentum? [4]

C3 A space station is in a circular orbit 150 km above Earth. Use Newtonian gravity to show that its orbital speed  $v$  is given by  $v^2 = GM/r$  (where  $M$  is the Earth's mass, of  $6.0 \times 10^{24}$  kg,  $r$  is the orbital radius; Earth's radius is 6380 km). [4]

Find the minimum energy per kilogram needed to launch a payload from Earth to the space station. [8]

A probe is then launched from the space station; find the minimum energy per kilogram needed to send the probe to deep space. [6]

Is there an overall energy advantage or disadvantage in sending a deep-space payload first to the space station and then launching it from there? Justify your answer. [6]

Discuss whether you would need more or less energy to send such a probe to the planet Mercury compared to the planet Jupiter. [6]

C4 Consider a frame  $S'$  moving at velocity  $v$  along the  $x$ -axis with respect to a stationary frame  $S$ . Motion  $u'_x$  as seen from  $S'$  will have an  $x'$  coordinate given by  $x' = u'_x t'$ .

Use this and the Lorentz transforms to show that the same motion viewed from  $S$  would have a speed

$$u_x = \frac{v + u'_x}{1 + vu'_x/c^2}. \quad [10]$$

As seen by one observer, Rocket A travelling at  $0.8c$  is being chased by Rocket B travelling at  $0.9c$ . As viewed from Rocket A, at what relative speed is Rocket B catching him? [10]

Rocket A fires a missile at  $0.1c$  (as he sees it) at Rocket B. At what speed does Rocket B see the missile approaching? [6]

Rocket B fires a laser beam at Rocket A; at what speed relative to Rocket A does this beam travel? [4]