EXAMINATION PAPER CONTAINS STUDENT'S ANSW

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The Handbook of Mathematics, Physics and Astronomy Data is provided

KEELE UNIVERSITY

EXAMINATIONS, 2011/12

Level I

Friday 25th May 2012, 09.30-11.30

PHYSICS/ASTROPHYSICS

PHY-10020

OSCILLATIONS AND WAVES

Candidates should attempt ALL of PARTS A and B, and ONE question from each of PARTS C and D. PARTS A and B should be answered on the exam paper; PARTS C and D should be answered in the examination booklet which should be attached to the exam paper at the end of the exam with a treasury tag. PART A yields 16% of the marks, PART B yields 24%, PART C yields 30%, PART D yields 30%.

Please do not write in the box below

A		C1		Total
B		C2		
		D1		
		D2		

NOT TO BE REMOVED FROM THE EXAMINATION HALL

PART A **Tick one box by the answer you judge to be cor**
(marks are not deducted for incorrect answers)

A1 Which one of the following functions is *not* a general solution to the equation $\ddot{x} = -\omega^2 x$? (ω, A, B, and ϕ_0 are all constants.)

☐ $x(t) = A\cos(\omega t + \phi_0)$ ☐ $x(t) = A\sin(\omega t) + B\cos(\omega t)$
☐ $x(t) = A\sin(\omega t) + A\sin\phi_0$ ☐ $x(t) = A\sin(\omega t - \phi_0)$ [1]

A2 A block of mass m attached to the end of a spring undergoes simple harmonic motion with an angular frequency $\omega = 12\ \mathrm{s}^{-1}$. A block of mass $4m$, attached to the same spring, would have an angular frequency of

☐ $3\ \mathrm{s}^{-1}$ ☐ $6\ \mathrm{s}^{-1}$ ☐ $12\ \mathrm{s}^{-1}$ ☐ $24\ \mathrm{s}^{-1}$ [1]

A3 A string of length L and a bob of mass m form a simple pendulum with period $T = 2$ s. The period of a pendulum with the same length but a bob of mass $4m$ is

☐ 0.5 s ☐ 1 s ☐ 2 s ☐ 4 s [1]

A4 The total mechanical energy of a particle in simple harmonic motion depends on the amplitude of the motion as

☐ $E_{\mathrm{tot}} \propto A^4$ ☐ $E_{\mathrm{tot}} \propto A^2$ ☐ $E_{\mathrm{tot}} \propto A$ ☐ $E_{\mathrm{tot}} \propto A^{1/2}$ [1]

A5 The motion of an object in the field of a conservative force is approximately simple harmonic near any position where the

☐ total energy is a local maximum.
☐ total energy is a local minimum.
☐ potential energy is a local maximum.
☐ potential energy is a local minimum. [1]

/Cont'd

A6 An oscillator with mass $m = 0.700$ kg and natural angular fre
$\omega_0 = 2.00\ \mathrm{s}^{-1}$ is damped by a force $F_{\mathrm{damp}} = -b\,\dot{x}$. The minimum va
of b that prevents oscillation about the equilibrium position is

- ☐ $b = 1.40\ \mathrm{kg\ s}^{-1}$ ☐ $b = 1.96\ \mathrm{kg\ s}^{-1}$
- ☐ $b = 2.80\ \mathrm{kg\ s}^{-1}$ ☐ $b = 4.00\ \mathrm{kg\ s}^{-1}$ [1]

A7 The frequency of a lightly damped harmonic oscillator is

- ☐ less than its natural frequency.
- ☐ greater than its natural frequency.
- ☐ equal to its natural frequency.
- ☐ independent of its natural frequency. [1]

A8 The steady-state displacement and velocity of an oscillator driven by a harmonic external force are

- ☐ in phase with each other. ☐ 45° out of phase.
- ☐ 90° out of phase. ☐ 180° out of phase. [1]

A9 A damped harmonic oscillator with natural angular frequency ω_0 is driven by a force $F(t) = F_0 \cos(\omega_e t)$. If $\omega_e = \omega_0$, then which one of the following statements about the steady state is *not true*?

- ☐ The rate of work done by the driving force is maximized.
- ☐ The rate of energy dissipation by the damping force is maximized.
- ☐ The velocity amplitude is maximized.
- ☐ The displacement amplitude is maximized. [1]

A10 In the scheme of analogies between electrical circuits and mechanical oscillators, the charge on the capacitor in a circuit corresponds to the

- ☐ mass ☐ displacement
- ☐ damping constant ☐ effective spring constant [1]

of a mechanical system.

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A11 Which *one* of the following functions could describe a wave tra in the $+x$ direction? (A, k, and ω are all positive constants.)

☐ $y(x,t) = A\sin(k^2x^2 - \omega^2t^2)$ ☐ $y(x,t) = A\,[\sin(kx) - \sin(\omega t)]$
☐ $y(x,t) = A\sin^2(kx - \omega t)$ ☐ $y(x,t) = A\sin(kx + \omega t)$ [1]

A12 The displacement of particles in a wave travelling along the x axis is given by $y(x,t) = 0.02\,e^{(8x+4t)}$, for x and y in metres and t in seconds. The particle speed at $x = 0$ at $t = 0$ is

☐ 2 m s^{-1} ☐ 0.5 m s^{-1}
☐ 0.16 m s^{-1} ☐ 0.08 m s^{-1} [1]

A13 The distance between two adjacent nodes in a standing wave of wavelength λ is

☐ $\lambda/(2\pi)$ ☐ $\lambda/2$ ☐ 2λ ☐ $2\pi\lambda$ [1]

A14 Two travelling harmonic waves combine to make the standing wave $y(x,t) = 0.02\,\sin(20\pi\,x)\cos(40\pi\,t)$ (for x and y in metres, and t in seconds). The wavelength of each of the travelling waves is

☐ $\lambda = 0.01$ m ☐ $\lambda = 0.05$ m
☐ $\lambda = 0.1$ m ☐ $\lambda = 0.2$ m [1]

A15 A string of length 24 cm vibrates in its third harmonic with both ends fixed. The wavelength of this standing wave is

☐ 8 cm ☐ 12 cm ☐ 16 cm ☐ 36 cm [1]

A16 Two sources emit travelling harmonic waves in phase with the same amplitude, wavelength, frequency, and intensity I_0. The maximum total intensity at points where the waves interfere constructively is

☐ $2\,I_0$ ☐ $2\,I_0^2$ ☐ $4\,I_0$ ☐ $4\,I_0^2$ [1]

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PART B Answer all EIGHT questions

B1 A particle is in simple harmonic motion with an amplitude of 5 cm. At time $t = 0$ it passes through its equilibrium position with a velocity of -12 cm s^{-1}. Calculate the period of the oscillation. [3]

B2 An object of mass $m = 0.080$ kg is in simple harmonic motion about $x = 0$ with angular frequency $\omega = 3.0$ s^{-1} and a total mechanical energy $E_{\text{tot}} = 0.0081$ J. Find the positions x at which the object has speed $|\dot{x}| = 0.36$ m s^{-1}. [3]

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B3 A particular oscillator of mass m [kg] has the equation of motio

$$\ddot{x} + \frac{0.40}{m}\dot{x} + 0.64\,x = 0 ,$$

where x is measured in metres and time in seconds. For what values of m is the oscillator overdamped? [3]

B4 A damped harmonic oscillator is driven by an external force, $F(t) = F_0\cos(\omega_e t)$. Briefly explain what is meant by the *transient* in the motion of such an oscillator, and write down the general form for the *steady-state* displacement as a function of time. [3]

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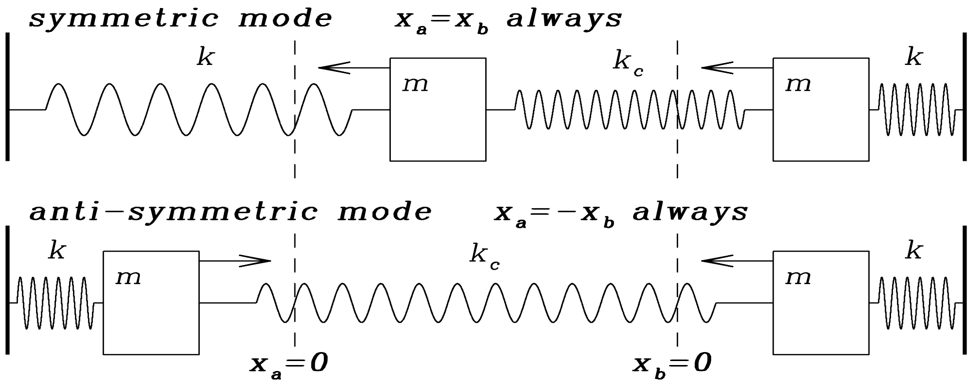
B5 Illustrated are the two normal modes of oscillation for a pair of identical blocks on identical springs, coupled by a third spring:



Write down a formula for the angular frequency of the symmetric mode. Is this frequency greater than or less than the frequency of the anti-symmetric mode? Justify your answers. [3]

B6 Ultrasound travels through human tissue as a harmonic wave with speed 1600 m s^{-1}, wavelength 4.5×10^{-4} m, and amplitude 1.8×10^{-9} m. Write the displacement $s(x, t)$ of molecules at depth x in the tissue at time t, assuming that $s = 0$ at $x = t = 0$. [3]

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B7 Verify that the function

$$y(x,t) \; = \; (2x-t)^3 + \cos(2x+t)$$

is a solution of the one-dimensional wave equation. [3]

B8 Two identical loudspeakers emit sound waves in phase, each with wavelength $\lambda = 2$ m and intensity $I_0 = 0.01$ W m^{-2}. What is the value of the total intensity at a point that is 30 m distant from one loudspeaker and 33 m from the other? (Justify your answer.) [3]

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PART C Answer ONE out of TWO questions

C1 (a) A 150-gram block is in simple harmonic motion on the end of a horizontal spring with force constant $k = 375\ \mathrm{N\ m^{-1}}$. At time $t = 0$, the block is 1.2 cm from its equilibrium position and has a speed of 36 cm $\mathrm{s^{-1}}$. Find

i. the acceleration of the block at $t = 0$; [2]

ii. the total mechanical energy of the block; [2]

iii. the amplitude of the oscillation; [2]

iv. the phase constant of the oscillation; [2]

v. one *time* at which the kinetic and potential energies of the block are equal. [4]

(b) The Sun can be crudely approximated as a sphere with uniform mass density, ρ_S. Consider a particle of mass m moving inside the Sun along a purely radial direction through the centre. Suppose that the only force acting on the particle is the force of gravity: $F(r) = -GmM(r)/r^2$, where $M(r)$ is the mass of the Sun within radius r from the centre.

i. Show that the particle experiences simple harmonic motion with angular frequency $\omega = \sqrt{4\pi G\rho_S/3}$. [10]

ii. The total mass of the Sun is $M_S = 1.99 \times 10^{30}$ kg and its radius is $R_S = 6.96 \times 10^8$ m. Thus, evaluate the period of simple harmonic motion in the Sun. [4]

iii. Calculate the maximum speed of a particle that is in simple harmonic motion in the Sun with an amplitude equal to R_S. Where in the Sun is this maximum speed achieved? [4]

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C2 The displacement x of a harmonic oscillator of mass m, which has a natural angular frequency ω_0 and is subjected to a damping force characterized by a constant b, may be given by one of the following three functions of time:

$$
\begin{aligned}
\text{(I)} \quad x(t) &= A_0\, e^{-bt/2m} \sin(\omega t + \phi_0) \quad \textit{with} \;\; \omega \equiv \sqrt{\omega_0^2 - b^2/4m^2} \\
\text{(II)} \quad x(t) &= e^{-bt/2m}\,(C_1\, t + C_2) \\
\text{(III)} \quad x(t) &= e^{-bt/2m}\left(B_1\, e^{qt} + B_2\, e^{-qt}\right) \quad \textit{with} \;\; q \equiv \sqrt{b^2/4m^2 - \omega_0^2}
\end{aligned}
$$

(a) Write down the equation of motion that is solved by any of the functions (I)–(III). State the physical meaning of each term in the equation of motion. [6]

(b) A block of mass $m = 0.250$ kg is attached to a spring with force constant $k = 1.44$ N m^{-1} and damping constant $b = 1.50$ kg s^{-1}. The block is in equilibrium at time $t = 0$, when it receives an impulse that gives it an initial velocity of -1.80 m s^{-1}.

i. Verify that this system is overdamped, and therefore state which one of equations (I), (II), or (III) above describes the motion of the block at $t > 0$. [4]

ii. Use the initial conditions given to determine both the position and the velocity of the block at any $t \geq 0$. [12]

iii. Sketch $x(t)$ for this system. Show on the same sketch and explain (without calculation) how $x(t)$ would change if the value of b were doubled while keeping all other parameters and initial conditions the same. [8]

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PART D Answer ONE out of TWO questions

D1 (a) Explain why a wave travelling with speed v in one dimension must depend on position x and time t either in the combination $(x - vt)$ or in the combination $(x + vt)$. [8]

(b) A string with uniform linear mass density μ is stretched along the x-axis and kept under a constant tension F. A transverse harmonic wave travels along the string, causing a displacement $y(x,t)$ at position x at time t. The kinetic energy per unit length in the string is given by

$$\frac{dK}{dx} = \frac{1}{2}\,\mu\left(\frac{\partial y}{\partial t}\right)^2$$

and the potential energy per unit length is given by

$$\frac{dU}{dx} = \frac{1}{2}\,F\left(\frac{\partial y}{\partial x}\right)^2 .$$

i. Write a general form for the wave function $y(x,t)$ in terms of the wavenumber and angular frequency. State what each of the partial derivatives $\partial y/\partial t$, $\partial^2 y/\partial t^2$, $\partial y/\partial x$, and $\partial^2 y/\partial x^2$ represents physically. [6]

ii. Show that any particle on the string undergoes simple harmonic motion. [4]

iii. Use the one-dimensional wave equation to derive the speed of the wave in terms of its angular frequency and wavenumber. [4]

iv. Given that $dK/dx = dU/dx$, infer a formula for the wave speed in terms of F and μ. [8]

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D2 (a) Two particular harmonic waves travelling along the x-axis c[...] bine to produce the standing wave

$$y(x,t) = 2A\,\sin(kx)\,\cos(\omega t)\ .$$

i. Write down the two travelling wave functions involved and verify explicitly that their superposition yields $y(x,t)$ as given. [6]

ii. Derive the allowed wavelengths λ_n if this is a resonant standing wave confined to $0 \le x \le L$, with $x = 0$ and $x = L$ both nodes. Find the positions of *all* nodes for the 4^{th} harmonic, and sketch this wave function at $t = 0$. [10]

(b) Two coherent sources, S_1 and S_2, emit harmonic waves with the same amplitude A, angular frequency ω, wavenumber k, and phase constant ϕ_0. These waves interfere at a point P, which is a distance x_1 from source S_1 and a distance x_2 from source S_2.

i. Show that the amplitude of the total wave at P is

$$A_{\text{tot}}(P) \;=\; 2A\,\cos\left[k(x_1 - x_2)/2\right]\ . \qquad [8]$$

ii. Hence, derive a general relation between the wavelength λ and the path difference $(x_1 - x_2)$ at points P where there is total destructive interference. [6]