The Handbook of Mathematics, Physics and Astronomy Data is provided

KEELE UNIVERSITY

EXAMINATIONS, 2010/11

Level III

Monday 17$^{\text{th}}$ January 2011, 16.00–18.00

PHYSICS/ASTROPHYSICS

PHY-30028

PHYSICS OF GALAXIES

Candidates should attempt to answer THREE questions.

A sheet of useful formulae and definitions can be found on the last page.

NOT TO BE REMOVED FROM THE EXAMINATION HALL

1. (a) The stellar mass density of the thin disc in the Milky Way

$$\rho(R, z) \;=\; \rho_0 \, e^{-R/h_R} \, e^{-|z|/h_z} \quad ,$$

where R is radius in the plane of the disc and z is height above the disc. The stellar disc mass within radius R is therefore

$$M(R) \;=\; 4\pi \, \rho_0 \, h_z \, h_R^2 \left[1 \;-\; e^{-R/h_R} \left(1 + \frac{R}{h_R} \right) \right] \quad .$$

i. The density of thin-disc stars in the Solar neighbourhood (i.e., at $R = 8$ kpc and $z = 0$) is $\rho \simeq 0.13 \; M_\odot \; \mathrm{pc}^{-3}$. Given that $h_R = 3.5$ kpc and $h_z = 300$ pc, calculate the value of ρ_0 and the total stellar mass of the thin disc. [10]

ii. Compare the mass of the thin disc within $R = 21$ kpc to the total enclosed mass implied by the observed circular speed at that radius, $V_c \simeq 220 \; \mathrm{km \; s^{-1}}$. Discuss whether the discrepancy can be attributed to the other stellar components of our Galaxy (consider the total masses and the density profiles of these other components). [30]

(b) The effective potential for a star with angular momentum L_z orbiting in the plane of an axisymmetric disc is

$$\Phi_{\mathrm{eff}}(R) \;=\; \Phi(R) + \frac{L_z^2}{2R^2} \quad .$$

Taking $\Phi(R) = V_0^2 \, \ln(R)$ in the plane of the Milky Way, show that $\partial^2 \Phi_{\mathrm{eff}} / \partial R^2 \;=\; 2V_0^2 / R^2$ for stars on circular orbits. What does this imply for the stability of the orbits? [30]

(c) Describe the motion in each of the radial, azimuthal, and vertical directions for a star on a nearly circular orbit close to the plane of the Milky Way disc. [30]

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2. The gravitational potential at radius r from the centre of a partic[...] spherical galaxy is given by

$$\Phi(r) \;=\; -\,\frac{G\mathcal{M}}{(r+a)} \;\;,$$

where $\mathcal{M}$ and a are constants.

(a) Without calculation, give an argument why $\mathcal{M}$ must be the total mass of the galaxy. [10]

(b) Show that the density profile, the enclosed-mass profile, and the half-mass radius of the galaxy are

i. $$\rho(r) \;=\; \frac{\mathcal{M}\,a}{2\pi\,r\,(r+a)^3} \;\;;$$ [20]

ii. $$M(r) \;=\; \mathcal{M}\,\frac{r^2}{(r+a)^2} \;\;;$$ [10]

iii. $$r_h \;\simeq\; 2.414\,a \;\;.$$ [10]

(c) The total potential energy, W, of the galaxy is given by

$$W = \frac{1}{2}\int_0^\infty \Phi(r)\,\rho(r)\,4\pi r^2\,dr \;\;.$$

i. Explain this equation in physical terms. [15]

ii. Show that
$$W \;\simeq\; -\,0.402\,\frac{G\mathcal{M}^2}{r_h} \;\;.$$ [15]

(d) The galaxy is observed to have a half-mass radius of $r_h = 10$ kpc and an average, one-dimensional internal velocity dispersion of $\sigma = 250$ km s^{-1}. Evaluate the total mass of the galaxy. [20]

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3. (a) In the context of accreting black holes, what is meant by the Eddington limit? Under what physical conditions might accretion at the Eddington limit be appropriate? [15]

(b) Consider a particle of mass m orbiting in an accretion disc around a black hole of mass M. The particle moves inwards from a radius $r + dr$ to a radius r from the centre of the black hole. Write down an expression for the change in the gravitational potential energy of the particle. [10]

(c) Half of the potential energy in part (b) is thermalised via friction in the disc and is radiated away, in the form of blackbody radiation. Thus, show that the temperature at radius r in the disc is equal to

$$T(r) = \left(\frac{GM\dot{M}}{8\pi\sigma_{\rm SB}\, r^3} \right)^{1/4} ,$$

where $\dot{M}$ is the mass accretion rate and $\sigma_{\rm SB}$ is the Stefan–Boltzmann constant. [30]

(d) A particular accretion disc extends down to 3 Schwarzschild radii from a non-rotating black hole of mass $10^8\ M_\odot$. Calculate the maximum temperature in this disc, assuming that the black hole accretes at the Eddington limit and that the efficiency of converting rest mass into energy is $\eta = 0.06$. [30]

(e) At what wavelength does the spectrum from this part of the disc peak? Many AGN also emit strongly in the infra–red and X-ray wavebands. State what mechanism(s) might account for the emission in these wavebands. [15]

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4. X-ray emitting gas in the galaxy group NGC 1550 has a temperat of $k_{\rm B}T = 1.5\,{\rm keV}$ out to a radius of 200 kpc. The gas is ionised hydrogen, for which $\mu = 0.5$.

   (a) Calculate the sound speed through the intergalactic medium of NGC 1550. Thus, what is the sound crossing time for the group? [15]

   (b) Estimate the mean line-of-sight velocity dispersion of the galaxies in NGC 1550. [10]

   (c) In hydrostatic equilibrium, the gas pressure P and the gas density ρ are connected to the gravitational potential Φ through

   $$\boldsymbol{\nabla} P = -\rho\, \boldsymbol{\nabla} \Phi \ .$$

      i. State why it is reasonable to assume hydrostatic equilibrium for NGC 1550. [5]

      ii. Given a spherically symmetric gas distribution, show that the mass of the group contained within radius r is

      $$M(r) = -\frac{r^2}{G\rho(r)} \frac{k_{\rm B}}{\mu m_{\rm p}} \frac{d(\rho T)}{dr} \ . \qquad [30]$$

   (d) The gas density in NGC 1550 is found to decrease as $\rho(r) \propto r^{-\beta}$, with $\beta = 1.1$, out to a radius of 200 kpc. Assuming that the gas is *isothermal*, calculate the mass of the NGC 1550 group within this radius. Give your answer in Solar units. [30]

   (e) The combined B-band luminosity of the galaxies in NGC 1550 is $L_B = 10^{11}\ L_\odot$. What is the mass-to-light ratio of the group? If the X-ray emitting gas has a mass of $10^{12}\ M_\odot$, what component of matter dominates the total mass of NGC 1550? [10]

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5. In the early Universe, a black hole of mass $M(t)$ grows by accretion, emitting radiation at a constant fraction f of the Eddington luminosity $L_{\rm Edd}$.

(a) Write down an equation that relates $\dot{M}$, the growth rate of the black hole mass, to f, $L_{\rm Edd}$, and the efficiency η of converting rest mass into energy. Hence, show that

$$\dot{M}(t) = \frac{f\,4\pi G m_{\rm p}}{\sigma_{\rm T}\, c\, \eta} M(t) \quad . \qquad [20]$$

(b) i. Solve the above equation for $M(t)$. [25]

ii. Calculate the Salpeter time, assuming $f = 0.3$ and $\eta = 0.06$. Give your answer in years. [15]

iii. Calculate the time required for a black hole to grow from an initial mass of 100 $M_\odot$ to a final mass of 10^9 $M_\odot$, assuming $f = 0.3$ and $\eta = 0.06$. Give your answer in years. [10]

(c) What is the total amount of energy radiated away during the growth phase of a black hole from 100 $M_\odot$ to 10^9 $M_\odot$? [10]

(d) It is known from the Sloan Digital Sky Survey that quasars containing black holes as massive as 10^9 $M_\odot$ exist at cosmological redshifts of $z = 6$, when the Universe was 0.95 Gyr old. Referring to your calculations above, discuss how feasible it may be to form such black holes by such redshifts. What factors might speed up the black hole growth? [20]

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Useful formulae and definitions

In spherical polar coordinates,

$$\nabla \Phi \;=\; \hat{\boldsymbol{r}}\,\frac{\partial \Phi}{\partial r} \;+\; \hat{\boldsymbol{\theta}}\,\frac{1}{r}\frac{\partial \Phi}{\partial \theta} \;+\; \hat{\boldsymbol{\phi}}\,\frac{1}{r\,\sin\theta}\frac{\partial \Phi}{\partial \phi}$$

$$\nabla^2 \Phi \;=\; \frac{1}{r^2}\,\frac{\partial}{\partial r}\left(r^2\,\frac{\partial \Phi}{\partial r}\right) \;+\; \frac{1}{r^2 \sin\theta}\,\frac{\partial}{\partial \theta}\left(\sin\theta\,\frac{\partial \Phi}{\partial \theta}\right) \;+\; \frac{1}{r^2 \sin^2\theta}\,\frac{\partial^2 \Phi}{\partial \phi^2}$$

For $n > 2$,

$$\int_0^\infty \frac{x}{(x+a)^n}\,dx \;=\; \frac{1}{(n-1)(n-2)\,a^{n-2}}$$

Eddington luminosity: $L_{\rm Edd} \;=\; \dfrac{4\pi G c M m_{\rm p}}{\sigma_{\rm T}}$

Stefan-Boltzmann law: $L \;=\; A \times \sigma_{\rm SB}\, T^4$

Schwarzschild radius: $R_{\rm S} \;=\; \dfrac{2GM}{c^2}$

Wien's law: $\lambda_{\rm max} \;=\; \dfrac{2.90 \times 10^{-3}\ \ {\rm m \cdot K}}{T}$

adiabatic sound speed: $c_s \;=\; \left(\dfrac{\gamma P}{\rho}\right)^{1/2}$