The Handbook of Mathematics, Physics and Astronomy Data is provided

KEELE UNIVERSITY

EXAMINATIONS, 2010/11

Level III

Tuesday 3rd May 2011, 13:00–15:00

PHYSICS/ASTROPHYSICS

PHY-30001

COSMOLOGY

Candidates should attempt to answer THREE questions.

NOT TO BE REMOVED FROM THE EXAMINATION HALL

1. Explain why, in a universe obeying the Cosmological Principle, expansion must obey Hubble's Law.

   By considering the energy of a galaxy obeying Hubble's law, show that the critical density of the universe today is given by

   $$\rho_{c,0} = \frac{3H_0^2}{8\pi G}$$

   (where the subscript zero denotes the current time). [25]

   Taking the Friedman equation

   $$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_{\rm m} - \frac{kc^2}{a^2} + \frac{8\pi G}{3}\rho_\Lambda,$$

   where $\rho_{\rm m}$ is the matter density and ρ_Λ the density owing to the cosmological constant, consider the dependances of $\rho_{\rm m}$ and ρ_Λ on the scale factor, and hence re-write the equation in terms of ρ_Λ and $\rho_{\rm m,0}$. [10]

   Further re-write the equation in terms of H_0 and the density parameters $\Omega_{\rm m,0}$ and $\Omega_{\Lambda,0}$. [10]

   Now consider a flat universe in which $\Omega_0 = \Omega_{\rm m,0} + \Omega_{\Lambda,0} = 1$, but which has $\Omega_{\Lambda,0}$ negative. Show that such a universe has a maximum size, and find $a_{\rm max}$ in terms of $\Omega_{\rm m,0}$ and $\Omega_{\Lambda,0}$. [20]

   Sketch the behaviour of the scale factor a over time in such a universe. [10]

/Cont'd

2. Use the Friedman equation

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{\Lambda}{3}$$

to show that the early, radiation-dominated universe expanded according to $a \propto t^{1/2}$. [25]

Hence show that the current age, t_0, of a radiation-dominated universe would be given by

$$t_0 = \frac{1}{2H_0}.$$

[20]

The redshift of a photon, emitted at λ_1 and seen at λ_0, is defined as

$$z = \frac{\lambda_0 - \lambda_1}{\lambda_1}.$$

Write down a relation between z and the values a_0 and a_1, and explain why this is justified. [20]

Hence show that, in such a universe, the time t_1 of emission of a photon of redshift z is given by

$$t_1 = \frac{1}{2H_0(z+1)^2}.$$

[20]

Sketch t_1 as a function of z. [15]

/Cont'd

3. (a) Produce an argument explaining why the early univer
have been radiation dominated. How would the temperature
changed as the universe expanded? [2

(b) At $\sim 10^{-8}$ secs after the Big Bang, the universe would have been a plasma of equal numbers of particles, anti-particles and photons. Explain the reason for this. [10]

(c) As the universe cooled the numbers of protons and neutrons became far fewer than the numbers of photons and neutrinos. Explain why. [10]

(d) As the universe cooled the equilibrium between neutrons and protons broke down, and the proton/neutron ratio became ~ 5. Explain why. Also explain the consequences of this for the end products of nucleosynthesis. [15]

(e) Why did nucleosynthesis in the early universe not occur earlier than ~ 100 secs after the Big Bang, and why did it not occur later than ~ 1000 secs? [15]

(f) The density of baryonic matter is thought to be $\Omega_B = 0.04$. Explain which observational results this value is based on. [15]

(g) Explain why we see a cosmic microwave background. [15]

/Cont'd

4. Write an outline account of the evidence for:

   (i) The Big Bang.

   (ii) Dark matter. [25]

   (iii) Dark energy. [25]

   (iv) The inflationary era. [25]

5. Write an account of the distance scale, outlining how we know the distances to astronomical objects, and discussing the assumptions and difficulties involved. Your answer should mention: parallax, standard candles, the Malmquist bias, Cepheid variables and supernovae. [100]