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KEELE UNIVERSITY

EXAMINATIONS, 2010/11

Level II

Friday 20$^{\text{th}}$ May 2011 09:30 - 11:30

PHYSICS/ASTROPHYSICS

PHY-20026

STATISTICAL MECHANICS AND SOLID STATE PHYSICS

Candidates should attempt to answer FOUR questions.

NOT TO BE REMOVED FROM THE EXAMINATION HALL

1. (a) Explain what is meant by

   i. reciprocal lattice vector

   ii. Miller indices [10]

   (b) A lattice has primitive translation vectors

$$
\begin{aligned}
\mathbf{a} &= a\,\mathbf{i} \\
\mathbf{b} &= a\,\mathbf{j} \\
\mathbf{c} &= 2a\,\mathbf{k}
\end{aligned}
$$

   where $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ are the usual cartesian unit vectors, and a is a constant.

   i. Determine the volume of the unit cell. [10]

   ii. Determine the reciprocal lattice vectors. [15]

   iii. Sketch the [111] and [001] planes. [20]

   iv. Determine the angle between the [111] and [001] planes. [40]

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2. A 1-dimensional chain consists of identical atoms each of
and separated by an equilibrium distance a.

(a) The n^{th} atom is displaced by an amount δ_n. Show that the net restoring force is

$$F = -\mu[2\delta_n - \delta_{n-1} - \delta_{n+1}] \, ,$$

where μ is the force constant. [10]

(b) Hence show that $\delta_n = \delta_0 \, e^{i(kna - \omega t)}$ is a solution of the equation of motion if

$$\omega = \pm 2 \left(\frac{\mu}{m}\right)^{1/2} \sin\left(\frac{ka}{2}\right)$$ [30]

where ω and k are respectively the angular frequency and wave-number of the vibration.

(c) Is this an 'acoustic' or an 'optical' vibration? Briefly explain your answer. [10]

(d) Show that the group velocity of sound in the long-wavelength limit is

$$v_{\mathrm{g}} = a \left(\frac{\mu}{m}\right)^{1/2}$$ [20]

(e) The 1-dimensional chain consists of copper atoms. The speed of sound in copper is 6420 m s^{-1}, and $a = 0.405$ nm. Determine whether a wave of angular frequency $\omega = 5 \times 10^{13}$ s^{-1} will propagate along the chain. [30]

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3. (a) Outline, in non-mathematical terms, the Drude theo electrical conductivity of metals.

(b) What are the successes and what are the shortcomings of th… Drude theory? [30]

(c) If τ is the mean time between electron and ion collisions in a metal, show that the Drude theory leads to Ohm's law for a conductor

$$\mathbf{J} = \frac{ne^2\tau}{m}\,\mathbf{E}$$

where $\mathbf{J}$ is the current density, $\mathbf{E}$ is the applied electric field, n is the number of electrons per unit volume, and e and m are the electron charge and mass respectively. [30]

(d) Copper has density 8 960 kg m^{-3}, resistivity 1.7×10^{-8} Ω m and valency 2. Estimate τ for copper. [20]

4. (a) Describe the mechanism underlying the Hall effect and derive an expression for the Hall coefficient R_{H} in terms of the number density of charge carriers n and their charge Q. [50]

(b) Hence explain why the Hall coefficient is expected to be negative for a metallic conductor. [10]

(c) A voltage difference of 100 mV is applied across the ends of a thin straight copper wire of length 1 m; a magnetic field of 10 T is applied perpendicular to the wire. Calculate

i. the number of electrons per m^3 [10]

ii. the Hall coefficient [10]

iii. the Hall field. [20]

[N.B. Copper has density 8 960 kg m^{-3}, resistivity 1.7×10^{-8} Ω m and valency 2.]

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5. The energy E of the interaction of a spin-$\frac{1}{2}$ paramagnet applied magnetic field B is given by $E = \pm\mu_B B$, where μ_B Bohr magneton and the $\pm$ sign denotes alignment anti-parallel (and parallel $(-)$ with the field.

(a) Write down an expression for the partition function of a system at temperature T that contains such paramagnets. [10]

(b) Write down an expression for the probability that each state is occupied. [10]

(c) Show further that the mean magnetic dipole moment is

$$\langle\mu\rangle = \mu_B \ \tanh\left(\frac{\mu_B B}{k_\mathrm{B}T}\right) \qquad [25]$$

(d) Hence write down an expression for the magnetization M for a material that contains n spin-$\frac{1}{2}$ paramagnets per unit volume. [5]

(e) i. Under what circumstances is the magnetization saturated? [10]

ii. Determine the saturation value of M for a material containing 3×10^{28} spin-$\frac{1}{2}$ paramagnets per m^3. [10]

(f) i. Under what circumstances is the magnetization negligible? [10]

ii. Show in this case that the magnetic susceptibility $\chi = M/B$ satisfies Curie's law:

$$\chi = \frac{n\mu_B^2}{k_\mathrm{B}T} \qquad [20]$$

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6. (a) Starting with the expression

$$n_i = A \exp\left[-\frac{E_i}{k_\mathrm{B}T}\right]$$

for the number of particles with energy E_i, write down an expression for the internal energy U in terms of temperature. [5]

(b) Hence show that the internal energy for a system consisting of N particles is

$$U = Nk_\mathrm{B}T^2 \frac{\partial \ln Z}{\partial T}$$

where Z is the partition function. [25]

(c) A system has two energy levels with energy $E_1 = 0$, $E_2 = \epsilon$. Assuming that the levels are non-degenerate,

i. show that the partition function for the system is

$$Z = 1 + \exp\left[-\frac{\epsilon}{k_\mathrm{B}T}\right]$$ [10]

ii. show that the internal energy is

$$U = \frac{\epsilon}{Z} \exp\left[-\frac{\epsilon}{k_\mathrm{B}T}\right]$$ [20]

iii. show further that the heat capacity of the system is

$$C = \frac{\epsilon^2}{Z^2 k_\mathrm{B}T^2} \exp\left[-\frac{\epsilon}{k_\mathrm{B}T}\right]$$ [40]