EXAMINATION PAPER CONTAINS STUDENT'S ANSWERS

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The Handbook of Mathematics, Physics and Astronomy Data is provided

KEELE UNIVERSITY

EXAMINATIONS, 2010/11

Level I

Monday 17th January 2011, 9:30–11:30

PHYSICS/ASTROPHYSICS

PHY-10022

MECHANICS, GRAVITY and RELATIVITY

Candidates should attempt ALL of PARTS A and B, and TWO questions from PART C. PARTS A and B should be answered on the exam paper; PART C should be answered in the examination booklet which should be attached to the exam paper at the end of the exam with a treasury tag. PART A yields 16% of the marks, PART B yields 24%, PART C yields 60%.

Please do not write in the box below

A		C1		Total
B		C2		
		C3		
		C4		

NOT TO BE REMOVED FROM THE EXAMINATION HALL

PART A **Tick one box by the answer you judge to be**

A1 The force acting on a body is equivalent to:

☐ dE/dx ☐ $E \times x$ ☐ m^2v ☐ dp/dx

A2 The units of momentum are equivalent to:

☐ $\mathrm{m^2\ s^{-2}}$ ☐ $\mathrm{kg\ m\ s^{-1}}$ ☐ kg m s ☐ $\mathrm{kg\ m^2\ s}$ [1]

A3 Given a force we can find the power by multiplying by:

☐ energy ☐ distance ☐ velocity ☐ mass [1]

A4 A 2-kg mass with speed 2 $\mathrm{m\,s^{-1}}$ collides with a stationary 1-kg mass. Assuming no loss of energy, the total kinetic energy afterwards is:

☐ 2 J ☐ 3 J ☐ 4 J ☐ 6 J [1]

A5 A rope slung between two trees has a weight in its center. For a given weight, a larger angle that the rope makes to the horizontal equates to a rope tension that is:

☐ larger ☐ smaller

☐ no different ☐ dependent on the rope length [1]

A6 A man pulls a sledge over level ground by a rope which makes an angle of 30° to the horizontal. The tension (force) in the rope is 30 N. In pulling the sledge over 8 m the work done is:

☐ 120 J ☐ 208 J ☐ 227 J ☐ 240 J [1]

A7 A canal boat is pulled by a horse; the tension in the rope is 800 N and the rope makes an angle of 20° to the direction of travel. To keep the boat straight, the rudder must exert a lateral force of:

☐ 752 N ☐ 153 N ☐ 851 N ☐ 274 N [1]

A8 A flywheel spins at 6π rad $\mathrm{s^{-1}}$. The revolutions per second is:

☐ 1 ☐ 2 ☐ 3 ☐ 6 [1]

/Cont'd

A9 If the angle turned by a wheel is described by $\theta = 30t$ [illegible]lar momentum is:

- ☐ increasing ☐ decreasing ☐ zero ☐ constant

A10 The angular momentum vector points:

- ☐ along the axis of rotation ☐ perpendicular to the torque
- ☐ in the plane of the body ☐ in the direction of θ [1]

A11 Angular acceleration, α, is equivalent to:

- ☐ ωt ☐ $\tau \times t$ ☐ dL/dt ☐ $d^2\theta/dt^2$ [1]

A12 Torque is equivalent to:

- ☐ $d\omega/d\theta$ ☐ $dE/d\theta$ ☐ $d\omega/dt$ ☐ $dP/d\theta$ [1]

A13 A body is in a circular orbit. Which of the following is not constant?

- ☐ its speed ☐ its energy
- ☐ its momentum ☐ its mass [1]

A14 Which of the following has the greatest mass?

- ☐ An infra-red photon ☐ An optical photon
- ☐ A UV photon ☐ An X-ray photon [1]

A15 The *proper* time is the time:

- ☐ for a muon to decay. ☐ after time dilation.
- ☐ measured in the rest frame. ☐ taken for light to travel. [1]

A16 Which of the following is invariant with velocity?:

- ☐ Δx ☐ Δt ☐ Δs ☐ mass [1]

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PART B Answer all EIGHT questions

B1 A car is travelling at a uniform speed. It then brakes to a stop with a constant deceleration. After being stationary for a period it then moves away at constant acceleration. Sketch graphs of the car's acceleration, speed and distance travelled. [3]

B2 A force of $F = 4x$ N, where x is distance, is applied to a particle over the range $0 < x < 5$ s. What is the work done? [3]

B3 If a skier skis down a slope of 35° (to the horizontal), in the absence of friction, what is his acceleration down the slope in terms of g? [3]

B4 Define the quantity angular momentum. [3]

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B5 One flywheel is made of a uniform disc. A second flywh all of its mass concentrated at its rim. If both are spun up by same torque, which is spinning faster, and why? [3]

B6 Planet A has a mass of 10^{25} kg and a radius of 10^{8} m. Planet B has a mass of 3×10^{25} kg and a radius of 2×10^{8} m. Which has the higher escape velocity? Justify your answer. [3]

B7 What is the momentum of a proton travelling at $0.8c$? [3]

B8 What is the total energy of an electron travelling at $0.8c$? [3]

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PART C Answer TWO out of FOUR questions

C1 A car weighs 1500 kg. Its engine gains power as it warms up and so the force generated is given by $F = 500\,t^{1/3}$ N. Given that the car starts from rest, find an expression for the car's velocity as a function of time. [7]

How far will the car have travelled after 30 secs? [7]

How fast will the car be moving at that point? [4]

What is the power being generated by the engine at that point? [4]

Evaluate the total work done by the engine up until that point. [4]

Again at that point, at what rate is the momentum of the car changing? [4]

C2 A circular disc of radius R and mass M has a uniform density. Starting from the definition of moment of inertia show that its moment of inertia about an axis running through the centre of the disc is:

$$I = \frac{1}{2}MR^2$$

[12]

Such a disc, with a radius of 2 m and a mass of 150 kg, is spun up by a rope coiled round the circumference of the disc; the end of the rope is pulled by a horse with a constant force of 100 N. After being pulled for 20 seconds, how fast is the disc rotating? [8]

At that point, what is the angular momentum of the disc and what is the energy stored in the disc? [6]

What is the work done by the horse on the disc over those 20 seconds? [4]

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C3 The vector

$$\boldsymbol{r} = A\cos(\omega t)\hat{\boldsymbol{\imath}} + A\sin(\omega t)\hat{\boldsymbol{\jmath}},$$

where A is a constant and t is time, traces out a circular orbit with an angular velocity of ω. By differentiating twice, show that the acceleration, $\boldsymbol{a}$, of a particle in that orbit is given by $\boldsymbol{a} = -\omega^2\boldsymbol{r}$. [10]

Use this to find the force keeping a particle of mass m in that orbit, and hence by comparing that to gravitational attraction show that the speed v of the orbit is given by $v^2 = GM/r$. [8]

Hence find the total energy needed to be given to a 250-kg payload to take it from Earth's surface and place it in an orbit 150 km above the surface. [Earth's radius is 6380 km; its mass is 6.0×10^{24} kg; ignore the Earth's rotation.] [12]

C4 A rocket (referred to as frame S') travels at $0.5c$ with respect to an asteroid (referred to as frame S), which is 20-m long as measured in its own frame. By considering the events of observing (simultaneously in S') the two ends of the asteroid, show by using the Lorentz transforms that the asteroid appears to be Lorentz contracted by a factor γ as seen from the rocket. [10]

Now suppose that two clocks, which are synchronised as judged from S, are placed at either end of the asteroid. The pilot in the rocket watches the two events of the second hand of each clock passing the minute mark. What does she measure the spatial distance between these two events to be? [10]

Now consider two consecutive ticks, 1 s apart in S, of the same clock. What is spatial and temporal separation of these events as seen from S'? [10]