EXAMINATION PAPER CONTAINS STUDENT'S ANSW

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The Handbook of Mathematics, Physics and Astronomy Data is provided

KEELE UNIVERSITY

EXAMINATIONS, 2010/11

Level I

Tuesday 24$^{\text{th}}$ May 2011, 09.30-11.30

PHYSICS/ASTROPHYSICS

PHY-10020

OSCILLATIONS AND WAVES

Candidates should attempt ALL of PARTS A and B, and TWO questions from PART C. PARTS A and B should be answered on the exam paper; PART C should be answered in the examination booklet which should be attached to the exam paper at the end of the exam with a treasury tag.
PART A yields 16% of the marks, PART B yields 24%, PART C yields 60%.

Please do not write in the box below

A		C1		Total
B		C2		
		C3		
		C4		

NOT TO BE REMOVED FROM THE EXAMINATION HALL

PART A **Tick one box by the answer you judge to be cor**
(marks are not deducted for incorrect answers)

A1 A block of mass m, hanging from a spring with force constant k and natural length L, oscillates purely in the vertical direction. The angular frequency of the oscillation is

☐ $\omega = \sqrt{k/m}$ ☐ $\omega = \sqrt{g/L}$
☐ $\omega = \sqrt{mg/k}$ ☐ $\omega = \sqrt{k/mg}$ [1]

where g is the acceleration due to gravity.

A2 A simple pendulum on the surface of the Earth has a period of 1 second. On the moon, where the acceleration due to gravity is 6 times lower, the period of the same pendulum would be

☐ 6 seconds ☐ $\frac{1}{6}$ seconds
☐ $\sqrt{6}$ seconds ☐ $\frac{1}{\sqrt{6}}$ seconds [1]

A3 An object is in simple harmonic motion about an equilibrium position, with an angular frequency of 3 s^{-1} and an amplitude of 0.4 m. The speed of the object at the equilibrium position is

☐ 3.6 m s^{-1} ☐ 2.4 m s^{-1} ☐ 1.2 m s^{-1} ☐ 0 [1]

A4 A particle is in simple harmonic motion. At how many times during one oscillation cycle are the kinetic and potential energies of the particle equal to each other?

☐ eight times ☐ four times ☐ two times ☐ one time [1]

A5 An oscillator with mass 300 g and natural angular frequency 6.00 s^{-1} is damped by a force $F_{\text{damp}} = -\gamma\dot{x}$. The critical damping constant is

☐ $\gamma = 1.80$ kg s^{-1} ☐ $\gamma = 2.55$ kg s^{-1}
☐ $\gamma = 3.60$ kg s^{-1} ☐ $\gamma = 10.8$ kg s^{-1} [1]

/Cont'd

A6 The amplitude of a particular underdamped oscillator dec[...] $A(t) \propto e^{-4t}$. The total mechanical energy of the oscillator depe[...] on time as

- ☐ $E_{\text{tot}}(t) \propto e^{-2t}$
- ☐ $E_{\text{tot}}(t) \propto e^{-4t}$
- ☐ $E_{\text{tot}}(t) \propto e^{-8t}$
- ☐ $E_{\text{tot}}(t) \propto e^{-16t^2}$ [1]

A7 The steady-state displacement and velocity of a forced harmonic oscillator are

- ☐ in phase with the external force
- ☐ 90° out of phase with the external force
- ☐ in phase with each other
- ☐ 90° out of phase with each other [1]

A8 A damped oscillator with natural angular frequency ω_0 is driven by an external force with angular frequency ω_e. The oscillator is shifted to a new equilibrium position if

- ☐ $\omega_e = 0$
- ☐ $\omega_e = \sqrt{2}\,\omega_0$
- ☐ $\omega_e = \omega_0$
- ☐ $\omega_e \gg \omega_0$ [1]

A9 In the scheme of analogies between electrical circuits and mechanical oscillators, the current in a circuit corresponds to the

- ☐ mass
- ☐ velocity
- ☐ displacement
- ☐ kinetic energy [1]

of a mechanical system.

A10 The wave described by $y(x,t) = e^{x-3t}\,(2x - 6t)^2$, with x and y in metres and t in seconds, propagates with a velocity of

- ☐ 3 m s^{-1} to the right
- ☐ 6 m s^{-1} to the right
- ☐ 3 m s^{-1} to the left
- ☐ 6 m s^{-1} to the left [1]

/Cont'd

A11 The wavenumber k of a harmonic wave is, in standard notation

☐ $k = m\omega^2$ ☐ $k = \omega v$ ☐ $k = 2\pi/\lambda$ ☐ $k = 2\pi\lambda$

A12 In the wave $y(x,t) = 0.03\,\sin(4x + 8t)$ (where x and y are in metres and t is in seconds), the particle velocity at $x = 0$ at $t = 0$ is

☐ 2 m s^{-1} ☐ 0.24 m s^{-1}

☐ 0.12 m s^{-1} ☐ 0 [1]

A13 Two travelling harmonic waves combine to produce the standing wave $y(x,t) = 0.01\,\sin(40x)\cos(60t)$ (for x and y in metres, and t in seconds). The amplitude of each of the travelling waves is

☐ $A = 0.005$ m ☐ $A = 0.01$ m

☐ $A = 0.02$ m ☐ $A = 0.1$ m [1]

A14 A string of length L with both ends fixed vibrates in its n^{th} harmonic. The distance between adjacent nodes on the string is

☐ $2L/n$ ☐ L/n ☐ nL ☐ $2nL$ [1]

A15 Two waves with the same intensity I_0 interfere at a point P in space. The maximum possible intensity of the total wave at P is

☐ $I_0\,/\,2$ ☐ I_0 ☐ $2\,I_0$ ☐ $4\,I_0$ [1]

A16 Interference patterns of the type seen in Young's double-slit experiment arise when waves emitted in phase by two sources arrive at a point in space

☐ from opposite directions

☐ having travelled different distances

☐ at different times

☐ with slightly different frequencies [1]

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PART B Answer all EIGHT questions

B1 A block of mass $m = 100$ g attached to a horizontal spring with $k = 40$ N m^{-1} has a displacement given by $x(t) = 0.05 \sin(\omega t)$ m. Calculate the velocity at time $t = T/2$, where T is the period of oscillation. [3]

B2 An object of mass $m = 0.4$ kg is in simple harmonic motion about $x = 0$ with angular frequency $\omega = 3$ s^{-1}. Its total mechanical energy is $E_{\text{tot}} = 4.5 \times 10^{-3}$ J. Find the speed of the object when its displacement is $x = 0.03$ m. [3]

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B3 A particular damped harmonic oscillator has the equation of motion

$$0.25\,\ddot{x} + \gamma\,\dot{x} + 0.16\,x = 0 \quad .$$

For what value(s) of the constant γ in this equation will the motion be overdamped? [3]

B4 A particular forced harmonic oscillator has the equation of motion

$$\ddot{x} + 0.16\,\dot{x} + 0.64\,x = 1.44\,\cos(\omega_e t) \quad .$$

Determine the value of ω_e that gives velocity resonance. Sketch the steady-state velocity amplitude as a function of ω_e in general. (You are *not* required to calculate any numerical values of the amplitude.) [3]

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B5 Give a sketch illustrating the two normal modes of oscillation for a coupled pair of identical blocks on identical springs. [3]

B6 A long string carries a transverse harmonic wave travelling in the negative-x direction with amplitude 2 cm, wavelength 60 cm, and frequency 440 Hz. The displacement of the string at $x = 0$ at $t = 0$ is $y = 0$. Write the wave function, $y(x, t)$. [3]

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B7 A travelling wave has the function

$$y(x,t) \;=\; 8x^2 + 6x + 8xt + 3t + 2t^2$$

for x in centimetres and t in seconds. Use the one-dimensional wave equation to find the phase speed of the wave. [3]

B8 A transverse wave travels at speed 330 m s^{-1} on a piano wire that has a total mass of 10 grams and a length of 64 cm. What is the tension in the wire? [3]

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PART C Answer TWO out of FOUR questions

C1 (a) A block of mass m on the end of a horizontal spring with force constant k is in simple harmonic motion about $x = 0$, with amplitude A.

i. Use the work-energy theorem to show that the potential energy of the block is $U = \frac{1}{2}kx^2$. [6]

ii. Sketch the potential and kinetic energies of the block as functions of the displacement x from equilibrium. [6]

(b) The potential energy of a simple pendulum with bob mass m and length L is

$$U(s) = mgL\left[1 - \cos\left(s/L\right)\right]$$

where s is the arc length from the bottom of the swing, and g is the acceleration due to gravity.

i. Find the values of s for which U is either a minimum or a maximum. What is the net force on the bob at each of these positions? Which of the positions is a stable equilibrium? [12]

ii. Infer formulae for the effective spring constant, and the angular frequency of small-amplitude oscillations, of a simple pendulum. [6]

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C2 (a) The displacement of an undriven, underdamped harmonic oscillator is given by

$$x(t) \;=\; A_0\, e^{-\gamma t/(2m)} \sin\left(\omega t + \phi_0\right) \quad ,$$

in which

$$\omega \;\equiv\; \sqrt{\omega_0^2 \,-\, \gamma^2/(4m^2)} \quad .$$

i. Sketch a representative $x(t)$ curve, indicating clearly all main physical features of the motion. [6]

ii. A block with $m = 0.4$ kg is attached to a damped spring having $k = 2.5$ N m^{-1} and $\gamma = 0.56$ kg s^{-1}. The block is in equilibrium at $t = 0$, when it receives an impulse giving it an initial velocity of $+0.6$ m s^{-1}.

A. Verify that this system is underdamped. [4]

B. Determine the displacement and the velocity of the block as functions of time for $t > 0$. [14]

(b) Explain what is meant by the *transient* and the *steady state* for the motion of an underdamped oscillator that is driven by an external force of the form $F(t) = F_0 \cos(\omega_e t)$. Write down the general form of the displacement $x(t)$ in the steady state. [6]

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C3 (a) Give an argument as to why a wave travelling with speed one dimension must depend on position x and time t only in one of the combinations $(x - vt)$ or $(x + vt)$. [6]

(b) The function

$$y(x,t) \;=\; A \sin\left[k(x - vt) + \phi_0\right]$$

describes a travelling harmonic wave. For such a wave:

i. The wavelength λ is defined as the smallest length such that $y(x+\lambda, t_0) = y(x, t_0)$ for any x at a fixed t_0. Use this to derive the standard relation between k and λ. [6]

ii. Show that y undergoes simple harmonic oscillation at any fixed position x in the wave. Thus, express the angular frequency ω of the wave in terms of k and v. [6]

(c) Consider the function

$$y(x,t) \;=\; 4\, e^{x-2t} \;-\; e^{3x-6t} \quad .$$

i. Verify that this function is a solution to the one-dimensional wave equation. [6]

ii. Show that

$$\partial^2 y/\partial t^2 \;+\; 8\;\partial y/\partial t \;+\; 12\; y \;=\; 0 \quad .$$

Thus, what kind of oscillation drives this wave? [6]

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C4 (a) The displacement of a string vibrating in its third harm[...] with both ends fixed is

$$y(x,t) = 0.02 \sin(0.2\pi x) \cos(25\pi t) \quad ,$$

where x and y are in centimetres and t is in seconds.

i. Calculate the wavelength of this standing wave, and the length of the string. [4]

ii. Find the positions of all nodes on the string, and sketch the wave at $t = 0$. [6]

(b) Two sources, S_1 and S_2, emit harmonic waves in phase with the same amplitude, frequency, and wavelength. These waves interfere at a point P, which is a distance x_1 from source S_1 and a distance x_2 from source S_2. Show that the total wave at P is

$$y_{\text{tot}}(P) = 2A \cos\left[\frac{k(x_1 - x_2)}{2}\right] \sin\left[\frac{k(x_1 + x_2)}{2} - \omega t + \phi_0\right] \quad . \qquad [6]$$

(c) Two identical speakers placed at $x = 0$ and $x = 60$ m emit sound in phase, with wavelength $\lambda = 2$ m. Use the equation for $y_{\text{tot}}(P)$ from part (b) to answer the following:

i. Find all positions x along the straight line between the speakers, at which the total volume is a maximum. [8]

ii. Calculate the intensity of the sound at $x = 30.25$ m, relative to the maximum possible intensity. [6]