The Handbook of Mathematics, Physics and Astronomy Data is provided

KEELE UNIVERSITY

EXAMINATIONS, 2009/10

Level III

Wednesday 28th April 2010, 13.00-15.00

PHYSICS/ASTROPHYSICS

PHY-30003

The Physics of Compact Objects

Candidates should attempt to answer THREE questions.

NOT TO BE REMOVED FROM THE EXAMINATION HALL

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1. (a) Explain what is meant by the term "complete degeneracy" when applied to a gas of fermions. [10]

(b) The density of particle momentum states in a gas can be written as $g(p) = 8\pi p^2/h^3$, where p is the particle momentum. Show that in a completely degenerate gas of fermions, the Fermi momentum is given by

$$p_F = \left(\frac{3h^3}{8\pi}\right)^{1/3} n^{1/3},$$

where n is the fermion number density. [20]

(c) A white dwarf star, composed entirely of carbon, has a mass of $1\,M_\odot$, a radius of 4000 km and an interior temperature of 10^7 K. Assuming a constant density:

i. Calculate the Fermi momentum of the electrons in the white dwarf. [15]

ii. Show that the electrons in the white dwarf are close to complete degeneracy. [15]

iii. Using an appropriate criterion for their momenta, calculate what fraction of the electrons can be considered relativistic. [15]

(d) In a real white dwarf, the density varies with radial distance from the stellar centre. Explain why this is and discuss how this changes the ideal equation of state in the star between its centre and its surface. [25]

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2. (a) Explain why a degenerate gas of fermions exerts a significant pressure e at very low temperatures. [10]

   (b) The equation of state inside a star containing degenerate fermions can be approximated by $P = A\rho^{\alpha}$, where P is the pressure, ρ is the mass density and A is some constant. Assuming the star has constant density, use the virial theorem to show that

$$R^{3\alpha-4} = \frac{5A}{G}\left(\frac{3}{4\pi}\right)^{\alpha-1} M^{\alpha-2}\,,$$

   where M and R are the mass and radius of the star. [30]

   (c) Explain why stars where $dM/d\rho$ is negative are unstable and, using the result from part (b), show that this corresponds to $\alpha < 4/3$. [30]

   (d) Sketch a mass-density diagram for degenerate stars and, labelling the axes *quantitatively*, indicate the Chandrasekhar mass, the regions occupied by stable white dwarfs, by stable neutron stars and the region where collapse to a black hole occurs. [30]

3. (a) Explain why both the thermal energy of electrons and the energy released by gravitational contraction can usually be neglected when discussing the cooling of white dwarf stars. [20]

   (b) The luminosity of a white dwarf star is given by $L = BMT^{7/2}$, where M is the mass of the star, T is its internal temperature and B is a numerical constant equal to 10^{-30} in S.I. units. Assuming a constant heat capacity for the ions in the white dwarf and stating any other assumptions that you make, show that the time τ taken for a white dwarf to cool from very high temperatures to a much lower temperature T, is given approximately by

$$\tau = \frac{3k_B}{5ABm_u}T^{-5/2}\,,$$

   where A is the atomic mass of the ions in the white dwarf. [30]

   (c) If the lowest luminosity white dwarf stars observed in the Galaxy have $L \simeq 10^{-5}L_{\odot}$ and $M \simeq 0.6M_{\odot}$ then estimate the age of the Galaxy, explaining your reasoning. [25]

   (d) The simple "Mestel" treatment of white dwarf cooling neglects crystallization. Describe crystallization and explain how it would modify your Galactic age estimate in part (c). [25]

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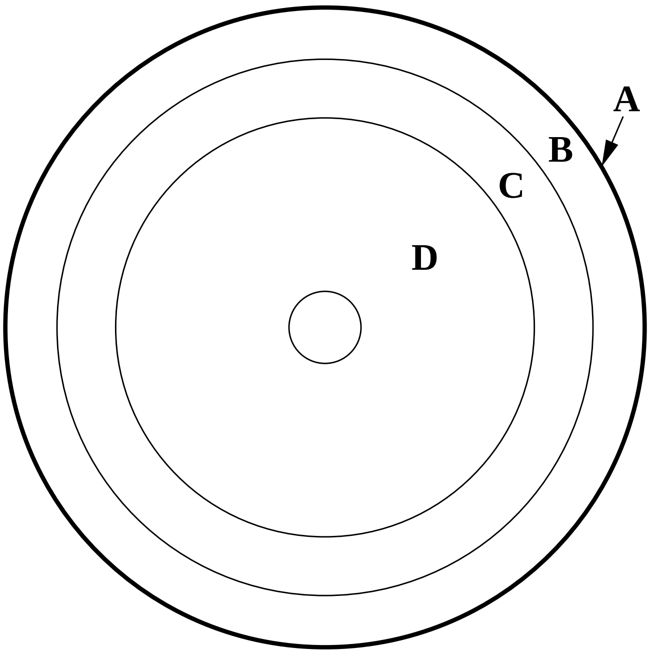
4. The diagram below represents the interior of a typical neutron star of mass 1.5



(a) Explain what is meant by an equation of state. [10]

(b) For each of the regions, labelled A–D, state what is the likely composition and density of the neutron star material. (NB: Do not discuss the presence or likely composition of any neutron star core region.) [20]

(c) For each of the regions, labelled A–D, discuss the nature of the equation of state, identify which particles are responsible for the majority of the pressure and describe how the pressure varies with mass density. [40]

(d) For each of the interfaces between regions A and B, between B and C and between C and D, explain why the increasing density causes changes in the nature of the neutron star material. [30]

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5. A high density region within a neutron star is composed of ideal, degenerate ga of neutrons, protons and electrons.

   (a) Explain why all the neutrons do not undergo beta-decay. [15]

   (b) Explain why the Fermi momenta of the protons and electrons are equal. [5]

   (c) Show that the Fermi energies of the neutrons, protons and electrons are related by

   $$E_{F,n} = E_{F,p} + E_{F,e}$$

   [30]

   (d) If the number density of protons in the gas is $10^{43}\,\mathrm{m}^{-3}$ then:

      i. Show that the protons are non-relativistic and that the electrons are ultra-relativistic. [20]

      ii. Hence show that the number density of neutrons is about 60 times larger than the number density of protons. [20]

      iii. Explain why neutrons at this number density are unlikely to behave as an ideal gas. [10]

[The Fermi momentum of a degenerate fermion gas is given by

$$p_F = \left(\frac{3h^3}{8\pi}\right)^{1/3} n^{1/3} ,$$

where n is the number density of fermions.]