The Handbook of Mathematics, Physics and Astronomy Data is provided

KEELE UNIVERSITY

EXAMINATIONS, 2009/10

Level III

Monday 11th January 2010, 13.00-15.00

PHYSICS/ASTROPHYSICS

PHY-30002

PHYSICS OF THE INTERSTELLAR MEDIUM

Candidates should attempt to answer THREE questions.

NOT TO BE REMOVED FROM THE EXAMINATION HALL

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1. For a linear molecule, the rotational energy levels are given by

$$E_J = \frac{\hbar^2}{2\mathcal{I}}\, J(J+1) \ ,$$

where $\mathcal{I}$ is the moment of inertia of the molecule and J $(= 0, 1, 2 \ldots)$ is the rotational quantum number.

(a) Define the *partition function* Z for a gas at temperature T. [10]

(b) Show that the partition function Z may be approximated by

$$Z = \frac{2\mathcal{I}\, k_{\mathrm{B}} T}{\hbar^2} \qquad [30]$$

(c) The interatomic distance for the $^{12}C^{16}O$ molecule is 1.128×10^{-10} m. Calculate the wavelength of the $J = 1 \rightarrow J = 0$ rotational transition. [10]

(d) A small molecular cloud with temperature 20 K lies at a distance of 500 pc. The $^{12}C^{16}O$ in the cloud emits radiation in the $J = 1 \rightarrow J = 0$ transition. Assuming that every photon emitted leaves the cloud, derive an expression for

i. the total power emitted, in terms of the number of $^{12}C^{16}O$ molecules in the cloud; [15]

ii. the flux observed at the Earth [5]

(e) The observed flux is 3×10^{-15} W m^{-2}; calculate the mass of $^{12}C^{16}O$ in the cloud. [30]

[N.B. The Einstein A coefficient for the $J = 1 \rightarrow J = 0$ transition in $^{12}C^{16}O$ is $A_{10} = 7.56 \times 10^{-8}$ s^{-1}]

2. Write an account of the observable effects of free electrons in various astrophysical environments. Your account should include relativistic and non-relativistic electrons, and those with thermal and non-thermal energy distributions. Answers should include a qualitive explanation of the physical principles involved. [100]

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3. A plane-polarized electromagnetic wave of circular frequency ω, pro
distance D through the interstellar medium, which contains $n_{\rm e}$ electrons m
magnetic field B; the direction of propagation is parallel to the magnetic fiel

(a) By considering the wave as a superposition of left- and right-handed circularly polarized waves, show that there arises a phase difference

$$(\mu_+ - \mu_-)\,\omega D/c\ ,$$

between the left- and right-handed waves, where μ_+ and μ_- correspond to the refractive indices for the left- and right-handed circularly polarized waves. [10]

(b) Hence explain why the plane of polarization rotates as the wave propagates. [5]

(c) Give an expression for the degree of rotation. [5]

(d) Assuming that $\omega_{\rm p}$ and $\omega_{\rm g}$ are both $\ll \omega$, show that

$$\mu_+ - \mu_- \simeq \frac{\omega_{\rm p}^2 \omega_{\rm g}}{\omega^3}\ .$$ [20]

(e) Show further that the plane of polarization of a plane-polarized wave of frequency ν (where $\omega = 2\pi\nu$) rotates through an angle

$$\Delta\phi = \frac{1}{\nu^2}\,\frac{1}{8\pi^2\epsilon_0}\,\frac{e^3}{m_{\rm e}^2 c}\int_{\rm Path} n_{\rm e}\,B\,dx\ .$$ [30]

(f) A Galactic radio source, which emits plane-polarized radiation, is at distance 1500 pc from the Earth. Along the path x from the source to the Earth the interstellar magnetic field B declines uniformly as

$$B = B_0 - \frac{x}{1500}\,B_1$$

where x is the path in pc, $B_0 = 3 \times 10^{-10}$ T and $B_1 = 5 \times 10^{-12}$ T. Also, for the first 1000 pc of the path, the electron density $n_{\rm e} = 10^4$ m^{-3}, but at 1000 pc jumps abruptly to 2×10^4 m^{-3} and maintains this value over the remainder of the path. Assuming that the radiation propagates along the magnetic field, calculate the Faraday rotation for radio waves of frequency $\nu = 5$ GHz. [30]

[N.B. The refractive index μ for a radiation of circular frequency ω is given by

$$\mu_+^2 = 1 - \frac{\omega_{\rm p}^2}{\omega(\omega + \omega_{\rm g})} \qquad \mu_-^2 = 1 - \frac{\omega_{\rm p}^2}{\omega(\omega - \omega_{\rm g})}\ ,$$

where the plasma frequency $\omega_{\rm p} = \sqrt{n_e e^2/\epsilon_0 m_{\rm e}}$ and the electron gyrofrequency $\omega_{\rm g} = eB/m_{\rm e}$.]

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4. (a) A hot star ionizises the gas in its environment. If the gas consis
hydrogen, explain why, in considering the balance between ionizati
recombination in the gas, only recombination to *excited* levels of hydro
need be considered. [10]

(b) A star is located in a uniform gas of pure hydrogen, with density $n_{\rm H}$ atoms $\rm m^{-3}$. The star emits S_* photons with energy > 13.6 eV. Show that the radius (the Strömgren radius) of the ionized region is

$$R = \left(\frac{3S_*}{4\pi n_{\rm H}^2 \beta}\right)^{1/3} ,$$

where β is the recombination coefficient to excited levels. [15]

(c) A B star emits 1.5×10^{49} photons $\rm s^{-1}$, of which 21% are in the Lyman continuum. A compact stellar cluster contains 10 B-type stars. How big is the region of ionized hydrogen around the cluster if the hydrogen number density is $3 \times 10^{8}\ \rm m^{-3}$? [25]

(d) The gas around the cluster actually contains a small amount of carbon, such that the relative abundance of C relative to H is 3×10^{-4} by number. Assuming that 10% of the photons ionize C, that the remainder ionize H, what are the relative sizes of the ionized H and C regions around the cluster? [50]

[N.B. The recombination coefficient to excited levels for H is $2.6\times10^{-19}\ \rm m^3\ s^{-1}$; the recombination coefficient to excited states for atomic carbon is $4.7\times10^{-19}\ \rm m^3\ s^{-1}$.]

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5. (a) A dust grain of radius a is located a distance r from a star ha nosity L. Assuming that the grain behaves like a black body show th equilibrium temperature T_{d} of the dust grain is

$$T_{\mathrm{d}} = \left(\frac{L}{16\pi r^2 \sigma} \right)^{1/4} ,$$

where σ is the Stefan-Boltzmann constant. [20]

(b) A population of spherical carbon particles is exposed to Lyman-α photons of wavelength 121.5 nm. Assuming that the particles behave like black bodies, determine the equilibrium temperature of the particles if the photon flux is $J = 10^{20}$ m^{-2} s^{-1}, deriving any expressions you need to use. [30]

(c) The population of dust grains in part (b) have radius $a = 5 \times 10^{-9}$ m. Can these grains actually be said to have an equilibrium temperature? Give physical reasons for your answer. [50]

[N.B. The density of carbon is 2000 kg m^{-3}. At low temperatures, the molar specific heat of carbon is

$$C_V = 234R \left(\frac{T_{\mathrm{d}}}{2000} \right)^3$$

where R is the gas constant.]