The Handbook of Mathematics, Physics and Astronomy Data is provided

KEELE UNIVERSITY

EXAMINATIONS, 2009/10

Level III

Tuesday, 4th May, 13:00–15:00

PHYSICS/ASTROPHYSICS

PHY-30001

COSMOLOGY

Candidates should attempt to answer THREE questions.

NOT TO BE REMOVED FROM THE EXAMINATION HALL

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1. Assume that in a spiral galaxy like ours the visible mass is distributed in a flattened disc such that enclosed visible mass increases as $M \propto r^2$ where r is the radius from the center.

(a) Use Newtonian gravity to predict the 'rotation curve' velocity as a function of r. Explain why an observed 'flat' rotation curve implies a massive dark-matter halo beyond the visible extent. [25]

(b) Considering the motion of galaxies as given by Hubble's law, equate kinetic and potential energy of such a galaxy to show that the critical density of the universe is given by $\rho_c = \frac{3H^2}{8\pi G}$. [25]

(c) Take the matter density parameter to be $\Omega_{\rm m} = 0.3$. Assuming that this matter is distributed entirely in spherical galaxy halos, each 30 kpc in radius, and that such galaxies are randomly distributed with a density of one per $\rm Mpc^3$, find the matter density within each galaxy halo. (Take $H_0 = 71\ \rm km\,s^{-1}\,Mpc^{-1}$.) [25]

(d) Now suppose that the 'dark energy' density parameter is $\Omega_\Lambda = 0.7$. Would you expect the dark energy to have a significant effect on the dynamics within a galaxy? Justify your answer. [25]

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2. Consider a flat universe consisting solely of material which has a pressure given by $P = w\epsilon$ where w is a constant and ϵ is the energy density.

(a) Use the fluid equation

$$\dot{\rho} = -3\,\frac{\dot{a}}{a}\left(\rho + \frac{P}{c^2}\right)$$

to find the dependence of the density ρ on the scale factor a for the case where $w = -1/2$. [25]

(b) Hence use the Friedman equation

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2}$$

to find the rate at which this universe expands with time. [25]

(c) What would be the age of such a universe in terms of the parameter H_0? [25]

(d) If the above universe was not flat, outline qualitatively the effect on the expansion rate. Include a sketch of a over time. [25]

3. The Friedman equation can be written (in the usual notation),

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{\Lambda}{3}.$$

(a) Show, justifying any assumptions you make, that a universe in the radiation-dominated era expands according to $a \propto t^{1/2}$. [25]

(b) Now consider a universe dominated by a positive cosmological constant Λ. Derive the expansion rate of such a universe. What is the value of the Hubble parameter in such a universe? [25]

(c) Now consider a flat universe in which the matter and Λ terms are equal. Sketch (justifying your reasons) the behaviour of a over a period before and after this time. [25]

(d) Further, consider a flat, matter-filled universe with a negative value of Λ. Outline the evolution of this universe, sketching (with justifications) a over time. [25]

You might find it helpful to refer to the acceleration equation

$$\frac{\ddot{a}}{a} = -\frac{4}{3}\pi G\left(\rho + \frac{3P}{c^2}\right) + \frac{\Lambda}{3}.$$

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4. Write a paragraph-length account of the following topics.

   (i) The emergence of the cosmic microwave background. [20]

   (ii) The Malmquist bias. [20]

   (iii) The equivalence principle. [20]

   (iv) Olbers's paradox. [20]

   (v) The products of Big-Bang nucleosynthesis. [20]

5. Write an account of the inflationary era. Discuss the nature of this era, outlining what is thought to have happened. Give an account of the various observational consequences for the universe today, as a result of inflation. [100]