KEELE UNIVERSITY

DEGREE EXAMINATIONS, 2010

Level 2 (PRINCIPAL COURSE)

Wednesday 13th January 2010, 13.00-15.00

PHYSICS/ASTROPHYSICS

PHY-20027

OPTICS AND THERMODYNAMICS

Candidates should attempt to answer FOUR questions.

The Handbook of Mathematics, Physics and Astronomy Data is provided

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1. (a) State and explain Huygens' and Fermat's Principles.

   (b) By using Fermat's principle, show that the angle of incidence is equal angle of reflection. [1

   (c) By using Fermat's principle, derive Snell's law of refraction. [20]

   (d) A ray of light is incident at an angle θ_i on the air interface with a glass block of thickness t and refractive index μ. Show that the lateral displacement d of the exit beam with respect to the incident beam is given by:

$$d = t\ \sin\theta_i \left[1 - \sqrt{\left(\frac{1-\sin^2\theta_i}{\mu^2-\sin^2\theta_i}\right)}\right] \qquad [50]$$

2. (a) Describe with a suitable diagram, the operation of a Michelson interferometer. [30]

   (b) In a Michelson interferometer the semi-silvered mirror is silvered on the front surface. Show that the condition for observing dark fringes is given by $2d\cos\theta = m\lambda$, where m is an integer, d is the effective separation of the mirrors, λ is the wavelength of the light and θ is the angle of observation. [30]

   (c) If the semi-silvered mirror in (b) is silvered on the back surface, show that the condition for observing the dark fringes is similar to that in (b). [10]

   (d) A Michelson interferometer is adjusted to observe white light fringes. The white light source is replaced by a sodium lamp and when one of the mirrors is moved by a distance d, the fringe system reaches minimum visibility. Assuming that the two wavelengths for sodium "D lines" are 589.6nm and 589.0nm, calculate a value for d. [30]

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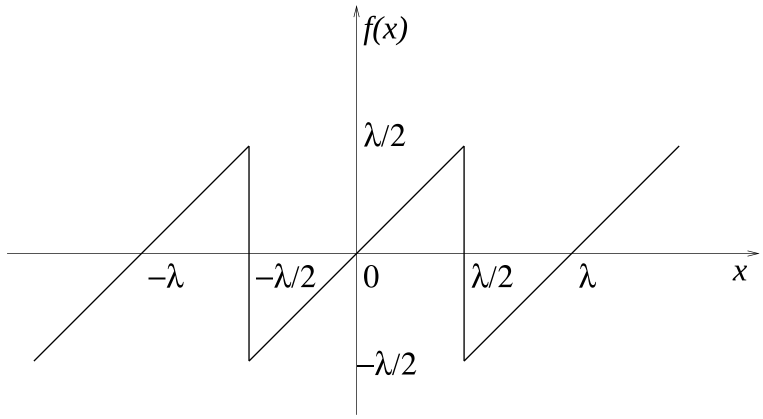
3. (a) Explain the terms *Fourier series* and *Fourier transform.*

(b) Compute the *Fourier series* of the saw-tooth wave shown below and by:

$$f(x) = x \qquad\qquad -\frac{\lambda}{2} \leq x < \frac{\lambda}{2}$$

and which is repeated to infinity.



[70]

(c) Without detailed calculation, explain the application of Fourier transforms to diffraction, illustrating your answer by reference to the single slit, including any diagrams as appropriate. [10]

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4. (a) Write down the First Law of Thermodynamics in differential form … what each of the terms means.

(b) Define the infinitesimal change of entropy of a system in terms of the temperature T of the system and the infinitesimal amount of energy $đQ$ exchanged with the system. [10]

(c) Show that the work done by a gas, which has pressure P, is $P\,dV$ when it expands by an infinitesimal volume dV. [15]

(d) Hence rewrite the First Law of Thermodynamics in terms of entropy, temperature, pressure and volume. [10]

(e) Calculate the change in entropy of the system for each of the following processes:

i. the melting of a piece of ice of mass 3 kg at 273.15 K in air at 273.16 K. [15]

ii. heating 3 ℓ of water from 20° C to 80° C at constant pressure. [15]

iii. the free, adiabatic (Joule) expansion of an ideal gas, resulting in doubling of the volume, and in which there is no change in the internal energy of the gas. [15]

[N.B. The latent heat of melting for ice is 334 kJ kg^{-1}. The specific heat at constant pressure for water is 4200 J kg^{-1} K^{-1}.]

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5. (a) Describe, with the aid of a suitable diagram, the operation of t... Cycle for an ideal gas.

(b) What is the change in the internal energy of the working substance that h... undergone a complete Carnot Cycle? Explain your answer. [10]

(c) Assuming that the efficiency of a reversible engine depends only on the tempertatures of the reservoirs, show that

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$$ [30]

(d) Hence show that the efficiency of a Carnot engine is

$$\eta = 1 - \frac{T_2}{T_1}$$

where T_2 is the temperature of the cold reservoir and T_1 that of the hot reservoir. [5]

(e) A Carnot Engine extracts heat from a reservoir at temperature 500 K. In each cycle the engine rejects 567 J of heat to a lower temperature reservoir. If the efficiency of the engine is 35%, calculate

i. the temperature of the cold reservoir; [5]

ii. the heat absorbed from the hot reservoir; [5]

iii. the work done on the working substance in each cycle. [5]

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6. (a) State the Clausius and Kelvin forms of the Second Law of Thermo[illegible]

[illegible]

(b) Show that each of these is equivalent to the other. [2 × [illegible]

(c) Carnot's theorem may be stated as follows:

> *no engine operating between two given heat reservoirs can be more efficient than a Carnot engine operating between the same two reservoirs.*

An inventor claims to have developed an engine that operates between two heat reservoirs at temperature 400 K and 300 K. In each cycle the engine extracts 5.5 kJ of heat from the high temperature reservoir, rejects 3.5 kJ to the low temperature reservoir, and does 2 kJ of work on the surroundings. Determine whether the engine is consistent with the laws of thermodynamics. [30]