The Handbook of Mathematics, Physics and Astronomy Data is provided

KEELE UNIVERSITY

EXAMINATIONS, 2009/10

Level II

Monday 18th January 2010, 09.30-11.30

PHYSICS/ASTROPHYSICS

PHY-20006

QUANTUM MECHANICS

Candidates should attempt to answer FOUR questions.

NOT TO BE REMOVED FROM THE EXAMINATION HALL

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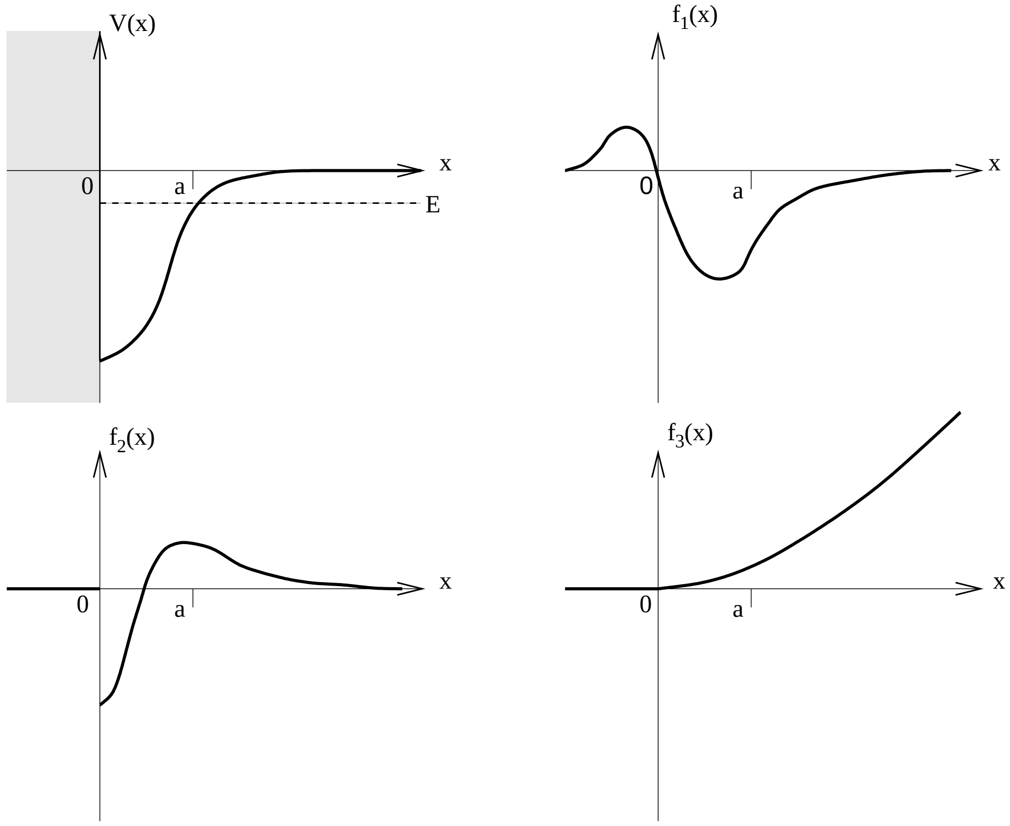
1. (a) State the interpretation of $|\Psi^\star\Psi|$, where Ψ is a wavefunction.

(b) The wavefunction for a particle has the following dependence on position, x.

$$\psi(x) = \begin{cases} \psi_1 = A\alpha\sin(x) & 0 < x < 3\pi/4 \\ \psi_2 = Ae^{-\beta x} & x \geq 3\pi/4 \end{cases}$$

Find the values of α and β for which ψ_1 and ψ_2 satisfy the requirements for an allowable wavefunction. [40]

(c) The diagram below shows a potential $V(x)$ experienced by a particle with energy $E < 0$, as indicated. The value of the potential for $x < 0$ is ∞. Also shown are three functions, f_1, f_2 and f_3.



i. For each function, give one reason why the function is not a valid solution of the time-independent Schrödinger equation for the particle in the potential $V(x)$. [3×10]

ii. Discuss briefly whether a valid solution, $\psi(x)$, of the time-independent Schrödinger equation should have a continuous first derivative $\frac{d\psi}{dx}$ at the point $x = 0$ in this case. [20]

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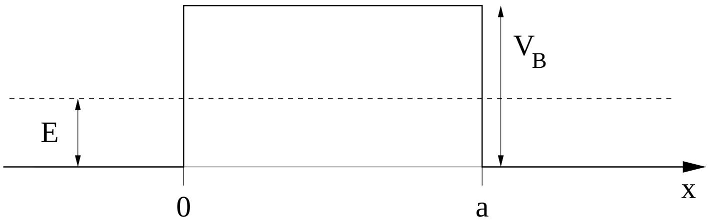
2. Consider a particle with mass m and energy E incident on a barrier with he
V_B and width a, such that $E < V_B$.



(a) Show that the energy eigenfunction

$$\psi_1 = A_I e^{ikx} + A_R e^{-ikx},$$

is a solution of the time-independent Schrödinger equation for the region $x < 0$ if $k^2 = 2mE/\hbar^2$. [30]

(b) What is the physical interpretation of the quantity $R = \frac{|A_R|^2}{|A_I|^2}$? [10]

(c) State an expression for the energy eigenfunction in the region $x > a$ in terms of the amplitude A_T. Explain your answer. [20]

(d) Give one example of a physical process that demonstrates or exploits the fact that $T = \frac{|A_T|^2}{|A_I|^2} > 0$. State the physical origin and approximate value of the barrier potential, V_B, for your example. [20]

(e) What is the limiting value of A_T in the case that $m \to \infty$? Explain your answer. [20]

[The time-independent Schrödinger equation in 1-dimension is:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)\,\psi = E\psi.$$
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3. A particle of mass m is located in the following "infinite square well" potenti

$$V(x) = \begin{cases} \infty & x < 0 \\ 0 & 0 \leq x \leq 2a \\ \infty & x > 2a \end{cases}$$

The general solution of the time-independent Schrödinger equation in the region $0 \leq x \leq 2a$ is

$$\psi(x) = A\sin(kx) + B\cos(kx),$$

where $k^2 = 2mE/\hbar^2$.

(a) Use the boundary conditions at $x = 0$ and $x = 2a$ to find the allowed values of B and k and, thus, the allowed values of the energy, E. [30]

(b) Calculate the value of the constant A in terms of a. Explain your method clearly. [30]

(c) Describe how the results above provide an explanation for the spectrum of black-body radiation emitted from a small hole in a hot cavity. [40]

[The time-independent Schrödinger equation in 1-dimension is:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)\,\psi = E\psi.$$

You may find the following standard integral to be useful.

$$\int \sin^2 x\,dx = \frac{1}{2}(x - \sin x\cos x) + C$$]

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4. The solutions ψ_n of the time-independent Schrödinger equation for a partic[le of] mass m in an harmonic oscillator potential $V(x) = \frac{1}{2}kx^2$ have energy E_n given by

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega, \qquad n = 0, 1, 2, \ldots,$$

where $\omega = \sqrt{k/m}$.

Consider the case of a ^{133}Cs ion trapped in an harmonic oscillator potential with $k = 1000\,\mathrm{Nm}^{-1}$ in a state with the wavefunction $\Psi = c_0\Psi_0 + c_1\Psi_1$.

(a) What is the interpretation of the constants c_0 and c_1 in the case of a wavefunction $\Psi = c_0\Psi_0 + c_1\Psi_1$? [10]

(b) For the case $c_1 = \frac{1}{\sqrt{2}}$ calculate

i. c_2

ii. the expectation value for the energy, $\langle E \rangle$, and

iii. the uncertainty in the measured energy, ΔE.

[40]

(c) Calculate the frequency of the photon emitted by the ^{133}Cs ion during the transition $n = 1 \rightarrow 0$. [20]

(d) Discuss whether the quantities $|\Psi^\star\Psi|$ and $|\Psi_1^\star\Psi_1|$ vary with time and, if they vary, give a timescale for the variation. [30]

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5. The energy eigenfunctions for an electron in a potential

$$V(r) = -\frac{e^2}{4\pi\epsilon_0 r}$$

have the form

$$\psi_{n,\ell,m_\ell}(r,\theta,\phi) = R_{n,\ell}(r)Y_{\ell,m_\ell}(\theta,\phi).$$

(a) What is the physical system represented by this potential? [10]

(b) State the value of the following quantities for the electron in the state ψ_{n,ℓ,m_ℓ}.

i. $\langle E \rangle$ [5]

ii. $\langle L^2 \rangle$ [5]

iii. $\langle L_z \rangle$ [5]

(c) The charge distribution for a system described by the potential $V(r)$ is observed to be spherically symmetric. What is the value of ℓ in this case? Explain your answer. [15]

(d) Outline the main features and results of the Stern-Gerlach experiment. Explain how this experiment shows that the eigenfunctions ψ_{n,ℓ,m_ℓ} do not give a complete description for the properties of an electron. [40]

(e) Describe briefly the consequences of the Stern-Gerlach experiment for the energy levels of the electron in the potential $V(r)$. [20]

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6. Consider a one-dimensional potential containing two identical, non-interact[...] particles, p and q, with the wavefunction $\Psi(x_p, x_q, t)$.

   (a) Give a physical interpretation of the quantity $|\Psi^\star \Psi| dx_p dx_q$. [20]

   (b) Use the fact that p and q are indistinguishable to show that $\Psi(x_p, x_q)$ must have definite exchange symmetry. [30]

   (c) Write down the antisymmetric energy eigenfunction ψ^{A} for the particles in terms of the energy eigenfunctions $\psi_\alpha(x)$ and $\psi_\beta(x)$ for a single particle in the same potential. [15]

   (d) What is the value of ψ^{A} in the case $\psi_\alpha(x) = \psi_\beta(x)$? Describe briefly the consequences of this observation for properties of multi-electron atoms. [35]

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