# EXAMINATION PAPER CONTAINS STUDENT'S ANSWERS

Please write your 8-digit student number here: ☐☐☐☐☐☐☐☐



**The Handbook of Mathematics, Physics and Astronomy Data is provided**

KEELE UNIVERSITY

EXAMINATIONS, 2009/10

Level I

Thursday 21st January 2010, 9:30–11:30

PHYSICS/ASTROPHYSICS

PHY-10022

MECHANICS, GRAVITY and RELATIVITY

Candidates should attempt ALL of PARTS A and B, and TWO questions from PART C. PARTS A and B should be answered on the exam paper; PART C should be answered in the examination booklet which should be attached to the exam paper at the end of the exam with a treasury tag.
PART A yields 16% of the marks, PART B yields 24%, PART C yields 60%. You are advised to divide your time in roughly these proportions.
Figures in brackets [ ] denote the marks allocated to the various parts of each question.

Please do not write in the box below

| | | | | |
|---|---|---|---|---|
| A | | C1 | | Total |
| B | | C2 | | |
| | | C3 | | |
| | | C4 | | |

**NOT TO BE REMOVED FROM THE EXAMINATION HALL**

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**PART A** TICK THE BOX BY THE ANSWER YOU JUDGE TO BE CORRECT (MARKS ARE NOT DEDUCTED FOR INCORRECT ANSWERS)



A1 The units of force are equivalent to:

☐ kg m $s^{-2}$ ☐ kg m $s^{-1}$ ☐ kg $m^2$ $s^{-2}$ ☐ kg $m^2$ s [1]

A2 A particle moving with distance $x = 2t^2$ has a velocity of:

☐ $8t$ ☐ $4t^2$ ☐ $4t$ ☐ 4 [1]

A3 A 1-kg hammer strikes a 10-g nail exerting a force of 80 N. The nail exerts a force on the hammer of:

☐ 0.8 N ☐ 8 N ☐ 80 N ☐ 100 N [1]

A4 The kinetic energy of a particle is changing. Which of the following statements *must* be false?

☐ $a$ is constant ☐ $a$ is decreasing ☐ $F$ is zero ☐ $\frac{dx}{dt}$ is decreasing [1]

A5 Particle $A$ has the same kinetic energy as particle $B$, but has twice the mass. Compared to the speed of $B$, its speed is:

☐ 2 ☐ $\sqrt{2}$ ☐ $1/\sqrt{2}$ ☐ 1/2 [1]

A6 The force acting on a body is equivalent to:

☐ $dE/dx$ ☐ $E \times x$ ☐ $m^2v$ ☐ $dp/dx$ [1]

A7 A 2-kg mass, travelling at 2 m $s^{-1}$, collides with a stationary mass of 1-kg. Assuming no loss of kinetic energy in the collision, the total kinetic energy of the masses after the collision is:

☐ 2 J ☐ 3 J ☐ 4 J ☐ 6 J [1]

A8 A machine is propelled by a force of 200 N which causes it to travel a distance of 5 m, in the same direction as the force, in 2 s. The average power is:

☐ 500 W ☐ 1000 W ☐ 1200 W ☐ 2000 W [1]

/Cont'd

A9 A flywheel rotates 1000 times per minute. Its angular velocity (in rad

- [ ] 2.7
- [ ] 17
- [ ] 105
- [ ] 6280

A10 Angular momentum is given by:

- [ ] $rm^2$
- [ ] $I\omega$
- [ ] $\int r^2 \, dm$
- [ ] $d\tau/d\theta$

[1]

A11 If the angle turned by a wheel is described by $\theta = 30t^2$ its angular velocity is:

- [ ] increasing
- [ ] decreasing
- [ ] zero
- [ ] constant

[1]

A12 Angular acceleration, $\alpha$, is equivalent to:

- [ ] $\omega t$
- [ ] $\tau \times t$
- [ ] $dL/dt$
- [ ] $d^2\theta/dt^2$

[1]

A13 Which of the following quantities is not necessarily conserved?:

- [ ] energy
- [ ] momentum
- [ ] angular momentum
- [ ] force

[1]

A14 Special Relativity applies only to:

- [ ] heavy bodies
- [ ] uniform relative motions
- [ ] accelerating bodies
- [ ] velocities near $c$

[1]

A15 A light beam travels at $c$ as seen by one observer. As seen by a rocket, travelling at $0.5\,c$ with respect to the first observer, the light beam appears to travel at:

- [ ] $0.5\,c$
- [ ] $c/\sqrt{0.75}$
- [ ] $c/(1 + 0.25)$
- [ ] $c$

[1]

A16 Newton's laws of motion remain valid in Special Relativity provided the mass is replaced by:

- [ ] $m_0$
- [ ] $\gamma m_0$
- [ ] $mc^2$
- [ ] $\beta m$

[1]

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B1 A car accelerates away from a traffic light, then coasts for a while, slowing gradually, and then applies the brakes to come to a halt at another traffic light. Sketch graphs of the car's acceleration, speed and distance travelled. [3]

B2 A force of $F = 3t$ is applied to a particle for a time $0 < t < 5$ s. What is the change in momentum? [3]

B3 A circular wheel, 2 m in radius, winds in a chain which pulls a coal truck up an incline. If the torque of the wheel is 3000 N m, what is the tension in the chain? [3]

B4 Define the quantity torque. [3]

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B5 A 4-m long rod of negligible mass is rotated about its center. Fixed
end of the rod are 3-kg masses. What is the moment of inertia?

B6 What is the gravitational potential 1000 km away from a mass of $10^{22}$ kg? [3]

B7 State the fundamental postulates of Special Relativity. [3]

B8 What is the total energy of an electron travelling at $0.8c$? [3]

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## PART C  ANSWER TWO OUT OF FOUR QUESTIONS

C1 The velocity of a particle of mass $m$ is given by $v = A\sin\left(\frac{\pi}{2}t\right)$ over the range $0 < t < 2$ where $A$ is a constant.

Find the distance travelled by the particle as a function of $t$. Similarly, find the acceleration of the particle. [10]

Sketch the velocity, distance and acceleration over the given time range (label all axes with values). [9]

Find expressions for the force on the particle and the power being exerted by the particle. [6]

What is the energy of the particle at $t = 1$? [5]

C2 A cylinder has a radius of 1 m and a total mass of 30 kg. The cylinder is made out of thin metal sheet such that all the mass can be taken as being at its rim. What is the moment of inertia of the cylinder as it rolls down a slope? [5]

Assuming that the cylinder rolls and does not slide, find the linear distance that the cylinder travels per revolution. Hence find a relation between the linear and rotational velocities of the cylinder. [5]

Find the kinetic energy associated with both the linear and the rotational motions. Is one greater than the other? [5]

The cylinder starts from rest and rolls downwards a vertical distance of 10 m. How fast is it rotating at this point? [Take $g = 9.8$ m s$^{-2}$.] [5]

What is the angular momentum of the cylinder at this point? [5]

Which direction is the angular momentum vector pointing? [2]

Is the cylinder exerting a torque on the Earth? Justify your answer. [3]

*/Cont'd*

C3 By equating gravitational attraction with the centripetal force show that a body in a circular orbit at a radius $r$ from a planet of mass $M$ must have a velocity given by

$$v^2 = \frac{GM}{r}.$$ [10]

If a space station orbits 100 km above the Earth find the total energy needed to be given to a 100-kg payload to land it on the space station. [Earth's radius is 6380 km; its mass is $6.0 \times 10^{24}$ kg; ignore the Earth's rotation.] [10]

If the space station fires thrusters, accelerating it in the same direction it is travelling, what changes in its orbit will result? [5]

If the space station then reverses the thrusters, and fires them against the direction in which it is travelling, describe what would happen. [5]

C4 Using the Lorentz transforms, prove that a time interval of $\tau$, as measured in its own frame $S$, appears to last for a time interval of $\gamma\tau$ when viewed from a moving frame $S'$. [12]

Cosmic-ray muons are created 5 km above the Earth's surface and travel downwards at a speed of $0.995c$. These particles are unstable and decay with a typical lifetime of $2.2 \times 10^{-6}$ s. How far does a muon typically travel, as seen from the muon's frame? Will a typical muon travel far enough to reach the Earth's surface? Justify and explain your answers. [10]

A detector stops 100 cosmic-ray muons. Given that the rest-mass of the muon is $1.88 \times 10^{-28}$ kg, what is the energy absorbed by the detector? [8]