EXAMINATION PAPER CONTAINS STUDENT'S ANSW

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The Handbook of Mathematics, Physics and Astronomy Data is provided

KEELE UNIVERSITY

EXAMINATIONS, 2009/10

Level I

Tuesday 18th May 2010, 13.00-15.00

PHYSICS/ASTROPHYSICS

PHY-10020

OSCILLATIONS AND WAVES

Candidates should attempt ALL of PARTS A and B, and TWO questions from PART C. PARTS A and B should be answered on the exam paper; PART C should be answered in the examination booklet which should be attached to the exam paper at the end of the exam with a treasury tag.
PART A yields 16% of the marks, PART B yields 24%, PART C yields 60%. You are advised to divide your time in roughly these proportions.
Figures in brackets [ ] denote the marks allocated to the various parts of each question.

Please do not write in the box below

A		C1		Total
B		C2		
		C3		
		C4		

NOT TO BE REMOVED FROM THE EXAMINATION HALL

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PART A TICK THE BOX BY THE ANSWER YOU JUDGE TO BE CORRECT (MARKS ARE NOT DEDUCTED FOR INCORRECT ANSWERS)

A1 The displacement and acceleration of a particle in simple harmonic motion are

- ☐ 90° out of phase
- ☐ 180° out of phase
- ☐ 270° out of phase
- ☐ 360° out of phase [1]

A2 The period of a simple pendulum with length L is

☐ $T = \sqrt{g/L}$ ☐ $T = \sqrt{L/g}$ ☐ $T = 2\pi\sqrt{g/L}$ ☐ $T = 2\pi\sqrt{L/g}$ [1]

A3 A particle is in simple harmonic motion with amplitude A. Its kinetic energy is a maximum at

☐ $x = 0$ ☐ $x = \pm A/2$ ☐ $x = \pm A/\sqrt{2}$ ☐ $x = \pm A$ [1]

A4 A block of mass 100 g on a spring with $k = 0.4$ N m^{-1} undergoes simple harmonic motion with an amplitude of 30 cm. Its total mechanical energy is

☐ 0.009 J ☐ 0.018 J ☐ 0.036 J ☐ 0.180 J [1]

A5 An oscillator with mass 200 g and natural angular frequency 3 s^{-1} is damped by a force $-\gamma\dot{x}$, in which $\gamma = 1.8$ kg s^{-1}. This system is

- ☐ overdamped
- ☐ critically damped
- ☐ underdamped
- ☐ resonantly damped [1]

A6 The period of an underdamped oscillator is

☐ equal to ☐ shorter than ☐ longer than ☐ independent of [1]

its natural period.

A7 In the steady state of a forced harmonic oscillator, the time-averaged mechanical energy

- ☐ is constant
- ☐ decays exponentially
- ☐ decays linearly
- ☐ grows exponentially [1]

A8 A harmonic oscillator with natural angular frequency ω_0 is driven by an external harmonic force with angular frequency ω_e. The power absorbed by the oscillator in its steady state is maximized if

☐ $\omega_e = 0$ ☐ $\omega_e \ll \omega_0$ ☐ $\omega_e = \omega_0$ ☐ $\omega_e \gg \omega_0$ [1]

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A9 Resistance in an electrical circuit is analogous to the

- ☐ mass
- ☐ driving force
- ☐ damping
- ☐ natural restoring force [1]

of a mechanical oscillator.

A10 A wave described by $y(x,t) = 0.02\ e^{-(2x+5t)^2/2}$ (with x and y in metres, and t in seconds) propagates at

- ☐ 5 m s^{-1} to the right
- ☐ 2.5 m s^{-1} to the right
- ☐ 5 m s^{-1} to the left
- ☐ 2.5 m s^{-1} to the left [1]

A11 A transverse wave travels at a speed of 160 m s^{-1} along a string with linear mass density mass 0.005 kg m^{-1}. The tension in the string is

☐ 0.004 N ☐ 0.8 N ☐ 128 N ☐ 32,000 N [1]

A12 In the wave $y(x,t) = 0.03\sin(4x - 8t)$ (x and y in metres and t in seconds), the particle acceleration at $x = t = 0$ is

☐ -1.92 m s^{-2} ☐ 1.92 m s^{-2} ☐ -0.24 m s^{-2} ☐ 0 [1]

A13 Two travelling harmonic waves combine to produce the standing wave $y(x,t) = 0.05\sin(12x)\cos(60t)$ (for x and y in metres, and t in seconds). The speed of the travelling waves is

☐ $v = 0.2$ m s^{-1} ☐ $v = 0.6$ m s^{-1} ☐ $v = 3$ m s^{-1} ☐ $v = 5$ m s^{-1} [1]

A14 A string of length 3 m vibrates in its fourth harmonic with both ends fixed. The wavelength of this standing wave is

☐ 0.75 m ☐ 1 m ☐ 1.5 m ☐ 2 m [1]

A15 Two waves with the same intensity I_0 interfere at a point P in space. The maximum possible intensity of the total wave at P is

☐ $I_0/2$ ☐ I_0 ☐ $2I_0$ ☐ $4I_0$ [1]

A16 The condition for minima in the intensity pattern from single-slit diffraction is

- ☐ $a\sin\theta = M\lambda, \quad M = 0, \pm1, \pm2, \ldots$
- ☐ $a\sin\theta = M\lambda, \quad M = \pm1, \pm2, \ldots$
- ☐ $a\sin\theta = \left(M + \frac{1}{2}\right)\lambda, \quad M = 0, \pm1, \pm2, \ldots$
- ☐ $a\sin\theta = \left(M + \frac{1}{2}\right)\lambda, \quad M = \pm1, \pm2, \ldots$ [1]

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PART B Answer all EIGHT questions

B1 A block of mass 40 g on a spring with $k = 16$ N undergoes simple harmonic motion with amplitude 5 cm. At time $t = 0$ it is displaced by $+3$ cm from its equilibrium position. What is the value of the phase constant? [3]

B2 An object of mass 0.3 kg is in simple harmonic motion about $x = 0$, with an amplitude of 4 cm. Its total mechanical energy is $E_{\text{tot}} = 0.054$ J. Find the period of the oscillation. [3]

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B3 A simple pendulum consists of a bob of mass $m = 150$ g at the end of string with length $L = 30$ cm. The force of air resistance on the bob is $F_{\text{air}} = -\gamma\,\dot{s}$, where $\dot{s}$ is the linear velocity and $\gamma = 0.03$ kg s^{-1}. Determine whether the pendulum is overdamped, critically damped, or underdamped. (The acceleration due to gravity is $g = 9.81$ m s^{-2}.) [3]

B4 Briefly explain what is meant by the *transient* stage and the *steady state* in the motion of a forced harmonic oscillator. [3]

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B5 Show that the function

$$y(x,t) \;=\; A\,e^{Bx-Ct}$$

is a solution of the one-dimensional wave equation (A, B, and C are arbitrary constants). [3]

B6 The displacement s of molecules in a sound wave travelling in the x direction is given by

$$s(x,t) \;=\; 7\times 10^{-8}\,\sin(5.3x - 1800t)$$

for x and s in metres and t in seconds. Calculate the wave speed and the maximum particle speed. [3]

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B7 A string of length 66 cm with both ends fixed vibrates in its third harmonic with a frequency of 784 Hz. What is the speed of travelling waves on this string? [3]

B8 Two identical speakers at positions $x = 0$ and $x = 300$ m face each other and emit sound waves in phase, each with intensity $I_0 = 0.01$ W m^{-2} and wavelength 4 m. What is the volume at the position $x = 151$ m between the two speakers? [3]

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PART C   Answer TWO out of FOUR questions

C1 (a) A particle of mass m undergoes simple harmonic motion about $x = 0$ with angular frequency ω and amplitude A.

i. Show that the potential energy of the particle is $U = \frac{1}{2}m\omega^2x^2$. [6]

ii. Use the general solution for $x(t)$ and the velocity $\dot{x}(t)$ to show that the total mechanical energy is $E_{\text{tot}} = \frac{1}{2}m\omega^2A^2$. [6]

iii. Sketch the kinetic and potential energies as functions of *position*. Identify the points x where kinetic energy equals potential energy. [6]

(b) A block of mass m hanging from a spring with constant k has a total (spring plus gravitational) potential energy of

$$U(y) = \tfrac{1}{2}ky^2 + mgy$$

where $y = 0$ is the position at which the spring is unstretched.

i. Find the position y_{eq} at which U is a *minimum*. What is the net force on the block at this position? [8]

ii. Using the formula $\omega^2 = (1/m)\left(d^2U/dy^2\right)_{y_{\text{eq}}}$, find the angular frequency and the period of simple harmonic motion of this block. [4]

C2 (a) The equation of motion of a damped harmonic oscillator is

$$m\ddot{x} + \gamma\dot{x} + m\omega_0^2x = 0 \ .$$

State the condition that determines whether an oscillator is underdamped, critically damped, or overdamped. Sketch a representative curve of displacement versus time for each case, indicating clearly all main physical features of the motion. [12]

(b) An oscillator driven by an external harmonic force with a tunable angular frequency ω_e has the equation of motion

$$m\ddot{x} + \gamma\dot{x} + m\omega_0^2x = F_0\cos(\omega_e t) \ .$$

i. What value of ω_e results in velocity resonance? [2]

ii. Assuming that the steady-state displacement at velocity resonance is given by $x(t) = A\sin(\omega_0 t)$, use the equation of motion to find A in terms of γ, ω_0, and F_0. [10]

iii. Show that the sum of the power input by the driving force, plus the power dissipated by the damping force, is exactly 0 in the steady state at velocity resonance. [6]

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C3 (a) A travelling wave has the equation

$$y(x,t) = A\ \sin(kx - \omega t + \phi_0) \quad .$$

i. Show that $k = 2\pi/\lambda$ implies $y(x+\lambda,\, t) = y(x,t)$ for any x and t. [4]

ii. Show that $y(x,t)$ satisfies the one-dimensional wave equation, and thus find the wave speed in terms of ω and k. [6]

iii. Show that there is simple harmonic oscillation about $y = 0$, with angular frequency ω, at any position x in the wave. [4]

(b) A standing wave is given by

$$y(x,t) \;=\; 2A\ \sin(kx)\cos(\omega t) \quad .$$

Derive the allowed wavelengths λ_n if the wave is confined to the region $0 \leq x \leq L$ and both endpoints $x = 0$ and $x = L$ are nodes. Sketch the wave functions of the first three harmonics at $t = 0$, with the positions of all nodes and antinodes clearly marked. [10]

(c) A quantum-mechanical particle of mass m is trapped in an infinite square well of width L. Obtain expressions for the momenta p_n and the kinetic energies E_n associated with the allowed de Broglie wavelengths of the particle. [6]

C4 (a) Two sources, S_1 and S_2, emit harmonic waves in phase with the same amplitude, frequency, and wavelength. The two waves interfere at a point P, which is located a distance x_1 from source S_1 and a distance x_2 from source S_2. Show that the total wave at P is

$$y_{\rm tot}(P) \;=\; 2A\ \cos\left[\frac{k(x_1 - x_2)}{2}\right]\sin\left[\frac{k(x_1 + x_2)}{2} - \omega t + \phi_0\right] \quad . \qquad [6]$$

(b) Use the expression for $y_{\rm tot}(P)$ from part (a) to derive relations between the path difference $(x_1 - x_2)$ and the wavelength λ for the case that the interference at P is

i. destructive. [6]

ii. constructive. [6]

(c) With the aid of a diagram, show that the path difference from two slits to a point on a distant screen in a Young's experiment is $d\ \sin\theta$ (define d and θ on the diagram). Hence, infer the standard expressions for the locations of bright and dark fringes in the intensity pattern on the screen. [12]