KEELE UNIVERSITY

DEGREE EXAMINATIONS, 2009

Level 3 (PRINCIPAL COURSE)

Friday, 1st May 2009, 9:30 - 11:30

PHYSICS/ASTROPHYSICS

PHY-30025

Life in the Universe

Candidates should attempt to answer THREE questions.

Tables of physical and mathematical data may be obtained from the invigilator.

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1. (a) Provide arguments in favour of the existence of conditions in the Solar System that are suitable to life, besides on Earth. [50]

   (b) Define the Drake equation, and discuss to what extent we are now able to quantify its individual components. [50]

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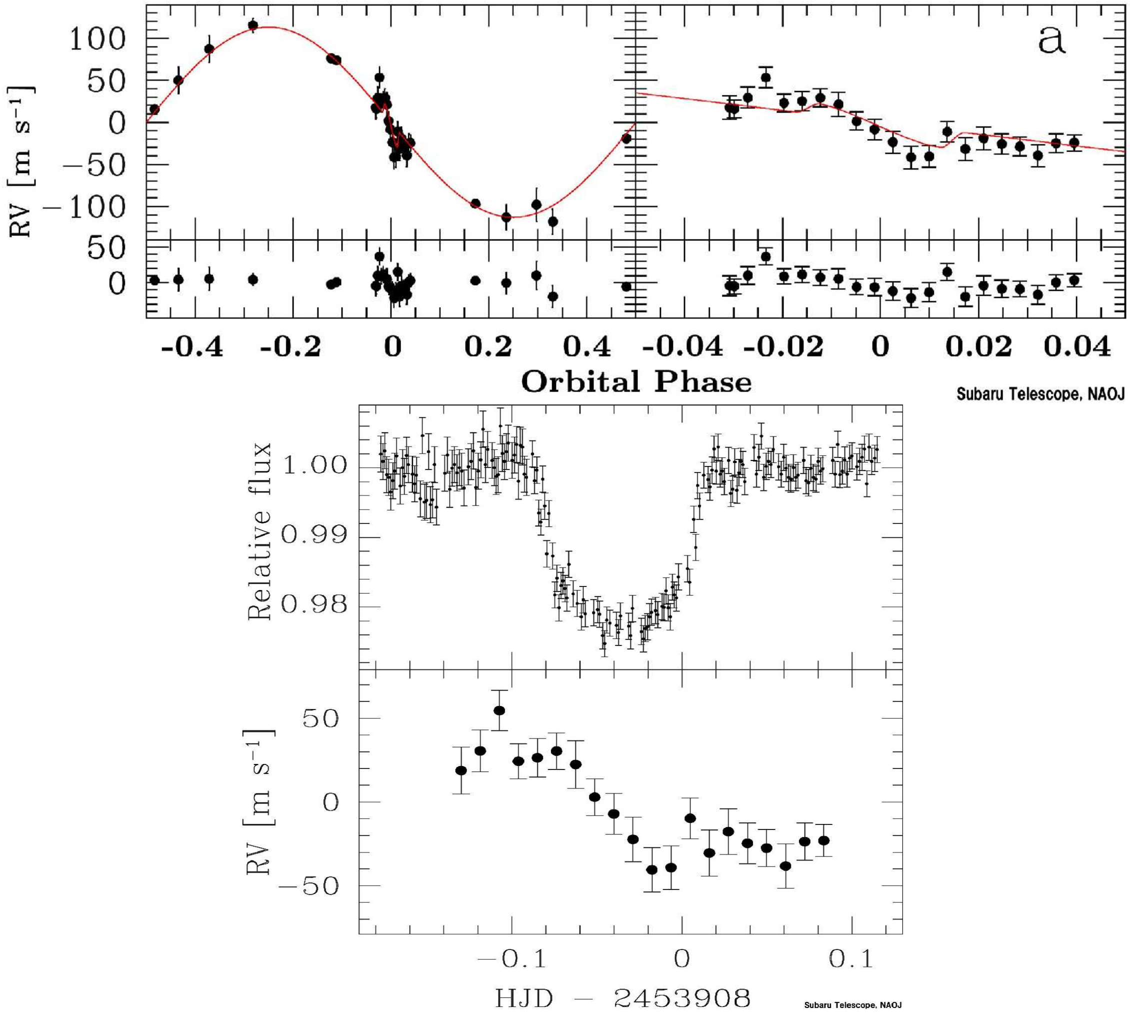

Figure 1: Velocity and brightness measurements of TrES-1, for use in question 2.

2. Exoplanet TrES-1b orbits a star with radius $R_\star = 0.82\ \mathrm{R}_\odot$ and mass $M_\star = 0.87\ \mathrm{M}_\odot$ in $P = 3.03$ days.

   (a) Use the measurements in Fig. 1 to obtain the mass of TrES-1b. [40]

   (b) Use the measurements in Fig. 1 to obtain the mean density of TrES-1b. [30]

   (c) Explain briefly the reason for the velocity variation during transit. [15]

   (d) Discuss briefly the suitability for life of TrES-1b. [15]

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3. (a) Derive the formula for the temperature T at the top of the atmosphere of a planet with bond albedo a, orbiting a star with luminosity $L_\star$ at a distance d:

$$T = \left(\frac{(1-a)L_\star}{16\pi\sigma\, d^2}\right)^{\frac{1}{4}} .$$ [30]

(b) Explain, in the context of planetary atmospheres, the Greenhouse effect. [30]

(c) For an isothermal atmosphere in hydrostatic equilibrium, derive the vertical pressure profile $P(z)$:

$$P(z) = P(0)\ \exp\left(-\frac{z}{H}\right),$$

and give a description of the scale height H in terms of the planetary and atmospheric properties. [40]

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4. Consider a uniform molecular cloud made up entirely of molecular hydrogen (H_2), with mass $M = 1000$ $M_\odot$ and radius $R = 1$ pc (1 pc = 206 265 AU). The cloud is initially in hydrostatic equilibrium.

The Jeans mass is given approximately as

$$M_J \sim 3c_s^3\, G^{-\frac{3}{2}}\, \rho^{-\frac{1}{2}},$$

where the sound speed $c_s = (kT/m)^{1/2}$, and ρ is the density.

(a) Show that $T \simeq 1000$ K and $M_J \sim 6M$. [30]

(b) The cloud cools rapidly to $T = 20$ K. Compute the Jeans mass in $M_\odot$, and describe what will happen next. [20]

(c) The cloud originally had a magnetic field of strength $B_0 = 1$ nT. Assume that the magnetic field strength is inversely proportional to the volume of the cloud. Assume $T = 20$ K. Compute the radius of the cloud at which the magnetic pressure is sufficient to halt further collapse of the cloud, and compare this with the original size of the cloud and the size of the Solar System. [30]

(d) Within the original cloud, cores of a Jeans mass typically rotate with a velocity $v = 1$ km s^{-1} at the edges of the cores. Compute the velocity at the edge of such core after it collapsed to a (uniform) core of the size computed in part (c). [20]

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5. A spacecraft with mass M_s approaches the Sun from a large distance, where its velocity was negligibly small. Around the time of closest approach, when the spacecraft is at one solar radius above the Sun's photosphere, the spacecraft is given a brief boost, increasing its velocity near-instantaneously by an amount Δv.

(a) Show that after the flyby, the spacecraft has a velocity

$$v_\infty \simeq ((2v_\mathrm{e} + \Delta v)\Delta v)^{\frac{1}{2}},$$

where v_e is the escape velocity from the Sun at closest approach. [30]

(b) To provide this boost, the spacecraft unfolds a sail at a grazing angle of $\alpha = 1°$ with respect to the Sun's radiation, for a duration of half an hour around the time of closest approach. The sail has a surface of $A = 10 \times 10\ \mathrm{km}^2$, and is made of ultra-thin foil with a surface density of $\Sigma = 0.01\ \mathrm{kg\ m}^{-2}$. The payload of the spacecraft has a mass of 10^6 kg. Show that the spacecraft undergoes an acceleration tangential to its path of

$$a = \frac{L_\odot A \sin\alpha}{2\pi r_\mathrm{p}^2\ M_\mathrm{s}\ c},$$

where r_p is the closest approach to the centre of mass of the Sun. [20]

(c) Compute the time it takes the spacecraft to reach the nearest star, Proxima Centauri, at a distance of 4.24 light years. [20]

(d) Discuss practical ways in which the spacecraft could use the sail to leave the Solar System with a velocity which exceeds that given in the equation for v_∞ above, and whether it could ever reach a velocity near the speed of light. [30]