KEELE UNIVERSITY

DEGREE EXAMINATIONS 2009

Level 3 (PRINCIPAL COURSE)

Thursday, 7th May 2009, 09:30 – 11:30

Astrophysics

PHY-30003

PHYSICS OF COMPACT OBJECTS

Candidates should attempt to answer THREE questions.

Tables of general physical and mathematical data may be obtained from the invigilator.

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1. (a) With reference to the occupation index of energy states for fermions, explain what is meant by a completely degenerate gas. [10]

(b) Assuming that the density of momentum eigenstates, $g(p)\,dp$ is given by

$$g(p)\,dp = \frac{8\pi p^2}{h^3}\,dp\,,$$

show that the Fermi momentum, p_F of a completely degenerate gas of fermions is

$$p_F = \left(\frac{3h^3}{8\pi}\right)^{1/3} n^{1/3}\,,$$

where n is the fermion number density. [20]

(c) The radius R of a neutron star composed entirely of completely degenerate neutrons and supported by *ideal*, *non-relativistic* degeneracy pressure is given by

$$R = 12.6\left(\frac{M}{M_\odot}\right)^{-1/3}\ \mathrm{km}\,,$$

where M is the neutron star mass.

(i) Obtain an expression for neutron star density as a function of mass. [10]

(ii) If neutron star interiors are typically at temperatures of 10^8 K, show that the assumption of completely degenerate neutrons is excellent for a $1\,M_\odot$ neutron star. [20]

(iii) Estimate for what masses the assumption of non-relativistic degeneracy breaks down. [20]

(iv) Estimate the separation of neutrons in a $1\,M_\odot$ neutron star and comment on whether the assumption of an ideal gas is likely to be valid. [20]

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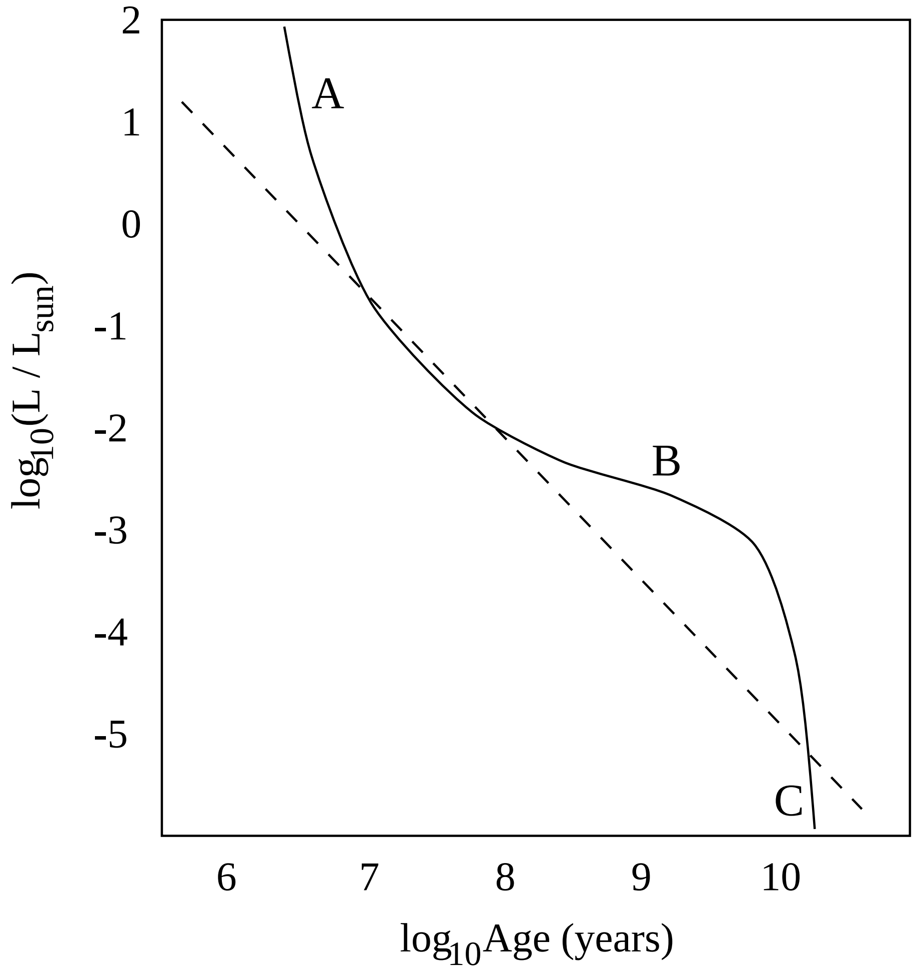
2. (a) Explain how electron degeneracy and its consequences influence the internal temperature structure and the total thermal energy of a white dwarf. [30]



(b) The solid line in the above plot shows a cooling curve (luminosity versus time in logarithmic units) for a $0.6\,M_\odot$ white dwarf, constructed from a numerical model. The straight dashed line represents the cooling curve derived using the Mestel approach, which assumes completely degenerate electrons, an ideal gas of carbon ions, and energy losses from a thin atmosphere in radiative equilibrium.

By referring to departures from these assumptions and changes in the internal structure of the white dwarf, explain in detail why the numerical models give different results in regions A, B and C. [50]

(c) Explain how a numerical cooling curve like this could be used to estimate the age of the Galaxy. [20]

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3. (a) What is meant by an *equation of state*? [10]

   (b) In the high-density neutron fluid region of a neutron star, explain why there are small fractions (by number) of protons and electrons and prove that, in equilibrium,

$$E_{F,n} = E_{F,e} + E_{F,p} \, ,$$

   where $E_{F,n}$, $E_{F,p}$ and $E_{F,e}$ are the Fermi energies of the neutrons, protons and electrons. [40]

   (c) Explain how this relationship could be used to derive an equation of state for the gas. [20]

   (d) Show that in such a gas, at extremely high densities, the ratio of protons to neutrons would be 1 to 8. [20]

   (e) Give *one* example of how additional types of particle might appear in the n-p-e gas at very high densities, and describe how this would modify the equation of state. [10]

[You may assume that $du/dn = E_F$, for a degenerate gas, where u and n are the gas energy density and fermion number density, and that the Fermi momentum is $(3h^3 n/8\pi)^{1/3}$]

4. (a) Explain why a degenerate gas of fermions exerts a pressure even at zero temperature. [10]

   (b) The pressure due to a cold, ultra-relativistic gas of non-interacting fermions is

$$P = \frac{hc}{8}\left(\frac{3}{\pi}\right)^{1/3} n^{4/3} \, ,$$

   where n is the fermion number density.

   (i) Using the virial theorem and assuming a star that has uniform density, show that there is a unique stellar mass that can be supported by ultra-relativistic degeneracy pressure. [30]

   (ii) If the star in question is a white dwarf made entirely of ionised carbon, calculate a numerical value for this limiting mass. [20]

   (c) Describe what is meant by inverse beta decay and discuss why it only affects the interiors of high-mass white dwarfs. Explain how inverse beta decay modifies the pressure of a relativistically degenerate electron gas and leads to instability at a smaller value of the limiting mass than found in part (b). [40]

[You may assume that the gravitational potential energy of a uniform sphere of mass M and radius R is $-3GM^2/5R$.]

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5. (a) In the hot, non-degenerate cores of massive stars, a major means of heat loss is via the URCA process.

(i) Explain what is meant by the URCA process. [10]

(ii) Explain why the URCA process is ineffective in neutron star interiors and describe a similar process that is more efficient. [30]

(b) An X-ray telescope detects a steady soft X-ray flux of $10^{-14}\,\mathrm{W\,m^{-2}}$ from a pulsar which has an age of 10^5 years and is at a distance of 1000 pc.

(i) Stating any assumptions you need to make, estimate the surface temperature of the pulsar. [20]

(ii) Discuss how your surface temperature could diagnose whether quark matter is present in neutron star interiors. [20]

(iii) Discuss two ways in which superfluid neutrons in the interior might modify the neutron star temperature at a given age. [20]