KEELE UNIVERSITY

DEGREE EXAMINATIONS 2009

Level 3 (PRINCIPAL COURSE)

Thursday, 30^{th} April, 9:30–11:30

ASTROPHYSICS/PHYSICS

PHY-30001

COSMOLOGY

Candidates should attempt THREE questions.

Tables of physical and mathematical data may be obtained from the invigilator.

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1. (a) By evaluating the kinetic and potential energies of a galaxy on the edge of a spherical region of universe show that the critical density is given by

$$\rho_c = \frac{3H^2}{8\pi G}.$$

[20]

(b) If the universe today (a_0 =1) has the critical density, and given that this is equivalent to an energy density, ϵ, of 8.3×10^{-10} J m^{-3}, what is the present value of the Hubble parameter? [10]

(c) Assume that the above energy density consists of contributions $\Omega_{\Lambda,0} = 0.7$, $\Omega_{\text{matter},0} = 0.3$ and $\Omega_{\text{radiation},0} = 0$. What will be the energy density of the universe when the universe has doubled in size from its current state ($a = 2$)? [20]

(d) What will be the value of the Hubble parameter at that time? [15]

(e) Suppose that, at that time, the Λ component is destined to evaporate into blackbody radiation. What would be the temperature just after that happened? ($\epsilon_{\text{rad}} = \frac{4\sigma}{c} T^4$) [10]

(f) What would be the value of the scale-factor when the universe returned to being matter dominated? [15]

(g) What would be the temperature at this epoch? [10]

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2. (a) Use the Friedman equation in the form

$$\dot{a}^2 = \frac{8}{3}\pi G\rho a^2 - kc^2$$

and the fact that the critical density is given by

$$\rho_c = \frac{3H^2}{8\pi G}$$

to show that

$$\Omega - 1 = \frac{kc^2}{a^2H^2}$$

where Ω is the density parameter. [35]

(b) Consider the inflationary era, when the universe was dominated by the zero-point energy density of the false vacuum, and find expressions for (1) the Hubble parameter, H, and (2) the scale factor, a, as a function of time. [30]

(c) Hence, discuss how the quantity $|\Omega - 1|$ changes through the inflationary era. [20]

(d) Outline the implications of the last result for the universe today. [15]

3. (a) Use the Friedman equation in the form

$$\frac{\dot{a}^2}{a^2} = \frac{8}{3}\pi G\rho - \frac{kc^2}{a^2} + \frac{\Lambda}{3}$$

to show that in a flat, matter-dominated universe the scale factor increases as

$$a \propto t^{2/3}.$$ [30]

(b) The universe currently has a temperature of 3K, whereas the temperature at matter–radiation decoupling was 3000K. Use this, while taking the current age of the universe as 10^{10} yrs, to estimate the age of the universe at decoupling. [20]

(c) What is the size of the largest region, S, across which light could have travelled by the time of decoupling? [10]

(d) To what size has the region S expanded to today? [10]

(e) Taking the distance of S from us as the light-travel distance since the emergence of the microwave background, what is the linear angle subtended by the region S as seen today? [10]

(f) Regions of the microwave background in opposite directions in the sky appear very similar. Discuss this result. [20]

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4. Explain why heavy free particles, such as the τ lepton, were common in the early universe, but not at later times. [10]

Consider the epochs when the universe had the following 5 temperatures.

(i) 3×10^{13} K (ii) 10^{11} K (iii) 4×10^{9} K (iv) 10^{8} K (v) 2000 K.

For each temperature, describe the main constituents of the universe and discuss the changes which occur as the universe cools to the next temperature, illustrating your answer with relevant particle or nuclear reactions. You may make use of the following data: [90]

Particle	Mass (MeV)	Temperature (K)
γ	0	0
ν_e, $\bar{\nu}_e$	$< 10^{-5}$	$< 10^{5}$
ν_μ, $\bar{\nu}_\mu$	< 0.27	$<3.1\times10^{9}$
ν_τ, $\bar{\nu}_\tau$	< 31	$<3.6\times10^{11}$
e^-, e^+	0.511	5.93×10^{9}
μ^-, μ^+	105.7	1.23×10^{12}
π^+, π^-	139.6	1.62×10^{12}
p, $\bar{p}$, n, $\bar{n}$	938.3, 939.6	1.06×10^{13}
τ^-, τ^+	1784	2.07×10^{13}
W^+, W^-	80220	9.31×10^{14}
Z^0	91187	1.06×10^{15}

Proton–Neutron mass difference ~ 1 MeV $\sim 10^{10}$ K
Deuteron binding energy ~ 2.2 MeV $\sim 2\times10^{10}$ K

5. Discuss, in about one paragraph each, the following topics.

(i) The Copernican principle. [20]

(ii) The Malmquist bias. [20]

(iii) The equivalence principle. [20]

(iv) The evidence for dark matter. [20]

(v) The Planck time. [20]