KEELE UNIVERSITY

DEGREE EXAMINATIONS 2009

Level 2 (PRINCIPAL COURSE)

Monday 18th May 2009, 13:00 – 15:00

PHYSICS

PHY-20026

STATISTICAL MECHANICS AND SOLID STATE PHYSICS

Candidates should attempt to answer FOUR questions,

Tables of physical and mathematical data may be obtained from the invigilator.

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1. The number of distinguishable particles n_i with energy ϵ_i is given by

$$n_i = A \exp\left[-\frac{\epsilon_i}{k_{\mathrm{B}}T}\right] .$$

   (a) Define the single particle partition function, Z_{sp}. [5]

   (b) Write down an expression for the total number of particles, N, in terms of Z_{sp}. [10]

   (c) Write down an expression for the internal energy U of the system in terms of n_i and ϵ_i. [10]

   (d) Hence show that the internal energy can be expressed as

$$U = Nk_{\mathrm{B}}T^2 \frac{\partial \ln Z_{\mathrm{sp}}}{\partial T}$$ [30]

   (e) For a certain system the partition function $Z_{\mathrm{sp}} = CT^{7/2}$, where C is a constant.

      i. Determine the internal energy of the system. [25]

      ii. Determine the specific heat of the system. [20]

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2. (a) Explain what is meant by a Schottky defect. [10]

(b) A crystal contains N ions. Explain why there are

$$\Omega = \frac{N!}{n!(N-n)!}$$

ways of placing n Schottky defects in the crystal. [20]

(c) To form each Schottky defect requires energy E_{S}, and the formation of n defects results in a change of free energy

$$\Delta F = \Delta E - T\,\Delta S \ ,$$

where ΔE is the energy required to form the defects and S is the entropy. Assuming that the most favourable value for n is that which makes ΔF a minimum, obtain an expression for the most favourable value of n in terms of N, E_{S} and T. [40]

(d) In NaBr the energy needed to form a defect is 1.2 eV. Calculate the number of defects in 1 kg-mole of NaBr at 650 K. [30]

[N.B. You may assume Stirling's approximation for large n: $\ln n! \simeq n \ln n - n$.]

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3. (a) Sketch the form of the Fermi-Dirac distribution for (i) a gas at $T = 0$ K and (ii) a gas at $T > 0$ K. Indicate on your plot the location of the Fermi energy. [30]

(b) Assuming that the density of states is given by

$$g(E)\,dE = \frac{V}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} E^{1/2}\,dE$$

in the usual notation, show that the Fermi energy is given by

$$E_{\mathrm{F}} = \frac{\hbar^2}{2m}\left(3\pi^2 n\right)^{2/3}$$

where n is the number of fermions per unit volume. [20]

(c) Solid gold has density 19 320 kg m^{-3} and valency 1. Calculate the Fermi energy for gold. [30]

(d) Determine whether or not the electrons in gold are relativistic. [10]

(e) Estimate the temperature of the *classical* gas for which the mean electron energy would be the same as your answer to part 3c. [10]

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4. (a) Describe the Hall effect and derive an expression for the Hall coefficient R_{H} in terms of the number density of charge carriers n and their charge Q. [50]

   (b) Hence explain why the Hall coefficient is expected to be negative. [10]

   (c) A thin straight magnesium wire of length 1 m is cooled to 100 K, at which the electrical conductivity is 1.1×10^{8} ohm^{-1} m^{-1} and the Hall coefficient is -7.8×10^{-11} m^{3} C^{-1}. A voltage difference of 10 mV is applied between the ends of the wire, and a magnetic field of 0.1 T is applied perpendicular to the wire. Calculate

      i. the number of charge carriers per m^{3}. [10]

      ii. the Hall field. [30]

5. Write an account of the development of our understanding of the specific heats of solids. Your account need not contain any mathematical detail but it should include the details of the assumptions behind the various theories, and an account of both the ionic and electronic contributions to the specific heat. [100]

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6. (a) In the context of semiconductors, state briefly what is meant by

   i. p-type material; [5]
   ii. n-type material; [5]
   iii. holes. [5]

(b) Sketch the energy-dependence of the density of states for a semiconductor with band-gap E_{g}. On your sketch indicate

   i. the conduction and valence bands; [5]
   ii. the band gap; [5]
   iii. the Fermi energy E_{F}; [5]
   iv. the energy states occupied by the conduction electrons and holes. [5]

(c) Assuming that the conduction electrons have energy E_{c}, and that holes have energy E_{v}, show that the concentration of conduction electrons and holes is

$$\begin{aligned} n &\simeq N_{\mathrm{c}} \exp[-(E_{\mathrm{c}} - E_{\mathrm{F}})/k_{\mathrm{B}}T] \\ p &\simeq N_{\mathrm{v}} \exp[-(E_{\mathrm{F}} - E_{\mathrm{v}})/k_{\mathrm{B}}T] \end{aligned}$$

respectively, where N_{c} and N_{v} are the density of states in the conduction and valence bands respectively. [35]

(d) Hence show that, in an intrinsic semiconductor, the concentration of conduction electrons is

$$n = [N_{\mathrm{c}}\, N_{\mathrm{v}}]^{1/2} \exp\left[-\frac{E_{\mathrm{g}}}{2k_{\mathrm{B}}T}\right]$$ [30]