KEELE UNIVERSITY

DEGREE EXAMINATIONS 2009

Level 2 (PRINCIPAL COURSE)

Tuesday 26th May, 16:00–18:00

ASTROPHYSICS

PHY-20002

STELLAR ASTROPHYSICS

Candidates should attempt FOUR questions.

Tables of physical and mathematical data may be obtained from the invigilator.

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1. The semi-empirical mass formula for the binding energy of a nucleus is

$$B(Z,N) = a_1 A - a_2 A^{2/3} - a_3 Z^2/A^{1/3} - a_4 (Z-N)^2/A + \delta(Z,A)$$

where Z, N and A are the number of protons, neutrons and nucleons respectively.

(a) Explain why this formula produces a maximum value near $A = 56$. [20]

(b) By referring to the formula, give an explanation for why, on a plot of Z versus N, the 'valley' of stable nuclei is curved. [15]

(c) ^{235}U can spontaneously decay by

$$^{235}_{92}\mathrm{U} \;\rightarrow\; ^{87}_{35}\mathrm{Br} \;+\; ^{145}_{57}\mathrm{La} \;+\; 3n.$$

Estimate the energy release in the reaction. [40]

(d) A 60-kg mass of ^{235}U is 'super-critical'. Estimate the energy released when it explodes. [10]

(e) Explain why the fission of ^{235}U produces free neutrons. [15]

[The coefficients of the semi-empirical mass formula are $a_1 = 15.8$, $a_2 = 17.8$, $a_3 = 0.697$, $a_4 = 23.3$; the δ term is $-12/\sqrt{A}$ for odd–odd nuclei, $+12/\sqrt{A}$ for even–even nuclei, and zero for odd–even nuclei; all values in MeV.]

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2. (a) Explain the physical origin of the 'Gamow energy', E_G, and state why it is important for the physics of stars. [$E_G = (\pi\alpha Z_A Z_B)^2\, 2m_r c^2$ where $\alpha = 1/137$ and $m_r = m_A m_B/(m_A + m_B)$.] [20]

(b) Calculate E_G for the two reactions: (1) $p\ +\ p\ \rightarrow\ d\ +\ e^+\ +\ \nu_e$ and (2) $p\ +\ {}^{12}\mathrm{C}\ \rightarrow\ {}^{13}\mathrm{N}\ +\ \gamma$. [20]

(c) Hence explain why the CNO cycle is the main mode of hydrogen burning in high-mass stars but not in low-mass stars. [10]

(d) The timescales for the reactions of the CNO cycle are:

$$\begin{aligned}
p + {}^{12}\mathrm{C} &\rightarrow {}^{13}\mathrm{N} + \gamma \quad (10^6 \text{ yrs})\\
{}^{13}\mathrm{N} &\rightarrow {}^{13}\mathrm{C} + e^+ + \nu_e \quad (14 \text{ min})\\
p + {}^{13}\mathrm{C} &\rightarrow {}^{14}\mathrm{N} + \gamma \quad (3 \times 10^5 \text{ yrs})\\
p + {}^{14}\mathrm{N} &\rightarrow {}^{15}\mathrm{O} + \gamma \quad (3 \times 10^8 \text{ yrs})\\
{}^{15}\mathrm{O} &\rightarrow {}^{15}\mathrm{N} + e^+ + \nu_e \quad (82 \text{ s})\\
p + {}^{15}\mathrm{N} &\rightarrow {}^{12}\mathrm{C} + {}^{4}\mathrm{He} \quad (10^4 \text{ yrs})
\end{aligned}$$

Using the above information (and assuming the CNO burning is occurring in equilibrium in a star), place the following in order of their relative abundance in the stellar core: ${}^{12}\mathrm{C}$, ${}^{13}\mathrm{N}$, ${}^{13}\mathrm{C}$, ${}^{14}\mathrm{N}$, ${}^{15}\mathrm{O}$, ${}^{15}\mathrm{N}$. [30]

(e) Estimate the abundance of ${}^{14}\mathrm{N}$ relative to ${}^{13}\mathrm{C}$. [20]

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3. (a) When a proton emits a π^+ meson, what particle remains? Draw a quark-level diagram illustrating the emission.

Similarly, draw a quark-level diagram for a neutron emitting a π^+. [30]

(b) The Δ^{++} is a heavy particle consisting of the quarks uuu. It decays in 10^{-20} seconds to a proton and a pion by the reaction

$$\Delta^{++} \rightarrow p + \pi^+ .$$

The Λ consists of the three quarks uds. It decays in 10^{-10} seconds by the reaction

$$\Lambda \rightarrow p + \pi^- .$$

By considering the reactions at the quark level, explain why the Λ lives 10^{10} times longer than the Δ^{++}. Draw a Feynman diagram for the reaction $\Lambda \rightarrow p + \pi^-$. [30]

(c) Draw a Feynman diagram illustrating the strong-force interaction between two quarks, labelling particles with their colour charge. [20]

(d) Why do we never see lone quarks or other coloured particles? [20]

[The u quark has a charge of +2/3; the d and s quarks have a charge of –1/3].

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4. (a) Considering shells within a star, write down the "conservation of mass" equation for $dm(r)/dr$, where $m(r)$ is the mass inside a radius r. [15]

(b) Assume that the pressure inside a star at radius r is given by

$$P(r) = \frac{P_c}{R}(R - r)$$

where P_c is the central pressure and R is the star's outer radius. Combine this and the equation of hydrostatic equilibrium,

$$\frac{dP}{dr} = \frac{-Gm(r)\rho(r)}{r^2},$$

to find an expression for $m(r)$. Hence show that $m(r) \propto r^{5/2}$. [40]

(c) Show, further, that the density in the star is given by

$$\rho(r) = \sqrt{\frac{5P_c}{8\pi G R r}}.$$

[20]

(d) Hence find an expression for the temperature, $T(r)$, given that the mean mass of the particles is $\bar{m}$ and assuming that the gas is non-degenerate. [15]

(e) Comment on whether your expressions are physically realistic. [10]

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5. Write an account explaining why a star leaves the main sequence; ascends the red-giant branch; moves onto the horizontal branch, and then ascends the asymptotic giant branch. The account should mention the Schönberg–Chandrasekhar limit, the Hertzsprung gap, the Hayashi forbidden zone, degeneracy, and the helium flash. At each stage, outline the mode of nuclear burning that is occurring. Explain, also, the changes in location on the H–R diagram. [100]

6. The plot below shows the relative abundance of the elements in the universe today. Explain the features of this plot in terms of nuclear physics and stellar evolution. In particular, explain the relatively high abundances of He, C, O and Fe, and the low abundances of $A = 5$–8 elements, $A \approx 40$ elements and $A \gg 56$ elements. [100]