KEELE UNIVERSITY

DEGREE EXAMINATIONS 2009

LEVEL 1 (PRINCIPAL COURSE)

Tuesday 19 May 2009, 13.00–15.00

PHYSICS/ASTROPHYSICS

MODULE PHY-10012

OSCILLATIONS AND WAVES

Candidates should attempt ALL of PARTS A and B, and TWO questions from PART C. PARTS A and B should be answered on the exam paper; PART C should be answered in the examination booklet which should be attached to the exam paper at the end of the exam with a treasury tag.

PART A yields 16% of the marks, PART B yields 24%, PART C yields 60%. You are advised to divide your time in roughly these proportions.

Figures in brackets [ ] denote the marks allocated to the various parts of each question. Tables of physical and mathematical data may be obtained from the invigilator.

Student Number ....

Please do not write in the box below

A		C1		Total
B		C2		
		C3		
		C4		

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PART A TICK THE BOX BY THE ANSWER YOU JUDGE TO BE CORRECT (MARKS ARE NOT DEDUCTED FOR INCORRECT ANSWERS)

A1 The velocity and acceleration of a simple harmonic oscillator are

- ☐ 180° out of phase
- ☐ 90° out of phase
- ☐ 45° out of phase
- ☐ in phase [1]

A2 A block of mass $m = 0.1$ kg on the end of a spring with $k = 0.4$ N m^{-1} is pulled 0.2 m away from equilibrium and released from rest. Its maximum acceleration is

- ☐ 0.08 m s^{-2}
- ☐ 0.4 m s^{-2}
- ☐ 0.8 m s^{-2}
- ☐ 3.2 m s^{-2} [1]

A3 When a particle in simple harmonic motion passes through its equilibrium position, its kinetic energy

- ☐ equals the total energy
- ☐ equals half the total energy
- ☐ equals the potential energy
- ☐ equals zero [1]

A4 The motion of a pendulum is approximately simple harmonic in the limit of

- ☐ short length
- ☐ small angles
- ☐ weak gravity
- ☐ low bob mass [1]

A5 An oscillator of mass 100 g and natural angular frequency 3 s^{-1} is subject to a damping force $-\gamma\dot{x}$, with $\gamma = 0.9$ kg s^{-1}. This system is

- ☐ underdamped
- ☐ critically damped
- ☐ resonantly damped
- ☐ overdamped [1]

A6 An oscillator with mass m and natural angular frequency ω_0 is damped by a force $F_{\text{damp}} = -\gamma\dot{x}$. Motion will be oscillatory, with a well-defined period, if

- ☐ $\gamma > 2m\omega_0$
- ☐ $\gamma = 2m\omega_0$
- ☐ $\gamma < 2m\omega_0$
- ☐ $\gamma < \sqrt{2}\,m\omega_0$ [1]

A7 A damped oscillator with natural angular frequency ω_0 is driven by an external harmonic force with angular frequency $\omega_e = \omega_0$. Which *one* of the following is *not* true in the steady state?

- ☐ the external force cancels the oscillator's natural restoring force
- ☐ the power input by the external force is a maximum
- ☐ the power dissipated by the damping force is a maximum
- ☐ the velocity of the oscillator is in phase with the external force [1]

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A8 Amplitude resonance is possible when a sinusoidally varying external force drives

- ☐ any oscillator
- ☐ a critically damped oscillator
- ☐ a very heavily damped oscillator
- ☐ a very lightly damped oscillator [1]

A9 Which *one* of the following functions could possibly represent a travelling wave? (A, B, and v are arbitrary numerical constants.)

- ☐ $y(x,t) = A\,(x^2 - v^2t^2)$
- ☐ $y(x,t) = Ax\big/(x+vt)$
- ☐ $y(x,t) = A\,\exp\left[-B\,(x-vt)^2\right]$
- ☐ $y(x,t) = A\,e^{B(x+vt)}\cos\left[B(x-vt)\right]$ [1]

A10 In the wave $y(x,t) = 0.2\cos(6x - 3t)$ [m], the particle speed at $x = t = 0$ is

- ☐ 0
- ☐ 0.5 m s^{-1}
- ☐ 0.6 m s^{-1}
- ☐ 1.2 m s^{-1} [1]

A11 A standing wave has the equation $y(x,t) = 0.4\,\sin(5x)\cos(2t)$, for x and y measured in metres and t in seconds. The speed of the component travelling waves is

- ☐ 2.5 m s^{-1}
- ☐ 2.0 m s^{-1}
- ☐ 0.8 m s^{-1}
- ☐ 0.4 m s^{-1} [1]

A12 A 3-metre long string is fixed at both ends. Which *one* of the following is *not* the wavelength of a normal mode on this string?

- ☐ 1/2 m
- ☐ 2/3 m
- ☐ 3/4 m
- ☐ 4/5 m [1]

A13 The power P transmitted by a wave is related to the wave amplitude A by

- ☐ $P \propto 1/A^2$
- ☐ $P \propto 1/A$
- ☐ $P \propto A$
- ☐ $P \propto A^2$ [1]

A14 Waves of the same frequency emitted by two sources in phase interfere destructively at points in space where the path difference from the sources is

- ☐ zero
- ☐ a half-integer number of wavelengths
- ☐ an integer number of wavelengths
- ☐ any non-zero amount [1]

A15 Waves with wavelength λ encounter an obstacle of size a. Diffraction effects will be greatest for

- ☐ $a \gg \lambda$
- ☐ $a \ll \lambda$
- ☐ $a \approx \lambda$
- ☐ $a = m\lambda/\sin\theta$ [1]

A16 The energy levels of a quantum-mechanical particle in an infinite square well obey

- ☐ $E_n \propto 1/n^2$
- ☐ $E_n \propto n^2$
- ☐ $E_n \propto 1/n$
- ☐ $E_n \propto n$ [1]

for n an integer.

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PART B ANSWER ALL EIGHT QUESTIONS

B1 A block of mass $m = 100$ g is attached to a horizontal spring with $k = 0.9$ N m^{-1}. The initial position and velocity of the block are $x(0) = 0.1$ m and $\dot{x}(0) = 0.3$ m s^{-1}. Find the displacement $x(t)$ as a function of time. [3]

B2 A particle of mass 100 g executes simple harmonic motion about $x = 0$ with frequency 0.5 Hz. At a certain instant, its kinetic energy is $K = 7.5 \times 10^{-3}$ J and its potential energy is $U = 5.0 \times 10^{-3}$ J. Find the amplitude of the oscillation and the maximum speed of the particle. [3]

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B3 A simple pendulum is made by attaching a ping-pong ball with a mass of 15 grams to a length of string with negligible mass. The force of air resistance on the ball is $F_{\text{air}} = -\gamma\dot{x}$, with $\gamma = 0.025$ kg s^{-1}. If the pendulum is critically damped by this force, what is the length of the string? [3]

B4 A damped oscillator with natural angular frequency ω_0 is driven by a force $F(t) = F_0 \cos(\omega_e t)$. Write the general expressions for the displacement and velocity as functions of time in the steady state. Sketch the velocity amplitude as a function of ω_e for a lightly damped system. Identify the value of ω_e that gives *velocity resonance.* [3]

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B5 A harmonic wave travelling in the $-x$ direction has a wavelength of 0.1 m, a frequency of 3400 Hz, and an amplitude of 10^{-6} m. Write the wave function $y(x,t)$, given that $y = 5 \times 10^{-7}$ m at $x = 0$ at $t = 0$. [3]

B6 Write the one-dimensional wave equation, and show that the function

$$y(x,t) = 2\,x^3 + 24\,x\,t^2$$

is a solution. What is the wave speed, if x and y are measured in metres and t in seconds? [3]

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B7 A violin string with linear mass density 0.003 kg m^{-1} is tuned by putting it under a tension of 580 N. It then vibrates at a frequency of 666 Hz in its fundamental mode. What is the length of the vibrating part of the string? [3]

B8 Monochromatic light illuminates a slit of width 0.3 mm, creating a diffraction pattern on a screen 2 m away from the slit. The first-order minima on the screen are located $\pm$3.7 mm on either side of the central intensity peak. What is the wavelength of the light? [3]

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PART C ANSWER TWO OUT OF FOUR QUESTIONS

C1 (a) Derive the period of a simple pendulum, $T = 2\pi\sqrt{L/g}$. Clearly state any approximations that are made. [12]

(b) A particle of mass m undergoes simple harmonic motion about $x = 0$, with angular frequency ω and amplitude A.

i. Show that the potential energy of the particle is $U = \frac{1}{2}m\omega^2x^2$. [6]

ii. Write the general expressions for $x(t)$ and $\dot{x}(t)$, and use these to show that the total mechanical energy is $E_{\text{tot}} = \frac{1}{2}m\omega^2A^2$. [6]

iii. Sketch the kinetic and potential energies as functions of *position.* [6]

C2 (a) Write the differential equation of motion for an oscillator of mass m and natural angular frequency ω_0, which is damped by a force $-\gamma\,\dot{x}$. [2]

(b) One possible solution of the equation of motion in part (a) is

$$x(t) \;=\; A_0\, e^{-\gamma t/(2m)} \sin(\omega\, t + \phi_0) \qquad \textit{with} \qquad \omega \;=\; \sqrt{\omega_0^2 - \frac{\gamma^2}{4m^2}} \;.$$

Name the type of damping described by this $x(t)$, and state the criterion that determines whether it applies to any particular system. Write expressions for the period and the amplitude of oscillation, and sketch a representative $x(t)$ curve that clearly illustrates these quantities. [8]

(c) A block of mass 0.4 kg on the end of a horizontal spring with $k = 2.5$ N m^{-1} experiences a friction force of the form $F_{\text{fric}} = -\gamma\,\dot{x}$, with $\gamma = 0.56$ kg s^{-1}. The block is in equilibrium at $t = 0$, when it receives an impulse giving it an initial velocity of $+0.6$ m s^{-1}.

i. Confirm that this system satisfies the criterion for $x(t)$ to have the form given in part (b). Determine the values of ω, A_0, and ϕ_0 for this system, and hence give its displacement and velocity as functions of time. [14]

ii. By what factor does the mechanical energy of this system decrease in one period? [6]

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C3 (a) Write the wave functions for two harmonic waves travelling along the x-axis in opposite directions but with identical amplitudes A, wavenumbers k, and angular frequencies ω, and both with $y = 0$ at $x = t = 0$. Show that the sum of these two waves is the standing wave,

$$y(x,t) = 2A \sin(kx) \cos(\omega t) \ . \qquad [6]$$

(b) Suppose the standing wave in part (a) is confined to $0 \leq x \leq L$. Derive the allowed wavelengths λ_n of the normal modes, and sketch the wave functions at $t = 0$ for the first three allowed harmonics, in the case that

i. $x = L$ is a *node.* [8]

ii. $x = L$ is an *antinode.* [8]

(c) Use results from either part (b)(i) or part (b)(ii), as appropriate, in the following.

i. A quantum-mechanical particle of mass m is in an infinite square well of width L. Obtain a formula for the energy levels of the particle, in terms of m, L, and n. *[Use $E = p^2/2m$.]* [4]

ii. A pipe of length $L = 39$ cm has one open end. The frequency of the second allowed harmonic for sound waves in the pipe is 660 Hz. Determine the speed of sound. [4]

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C4 (a) Two sources, S_1 and S_2, emit harmonic waves in phase with the same frequency. At a point P located a distance x_1 from source S_1 and a distance x_2 from source S_2, the sum of the waves is

$$y = 2A \cos\left[\frac{k\,(x_1 - x_2)}{2}\right] \sin\left[\frac{k\,(x_1 + x_2)}{2} + \omega\, t\right] \quad .$$

Use this expression to derive relations between the path difference and the wavelength, λ, for the case that the interference at P is

i. destructive, giving $y = 0$ at any time. [5]

ii. constructive, giving the maximum possible $|y|$ at any time. [5]

(b) With the aid of a clearly labelled diagram, show that the path difference from two slits to a point on a distant screen in a Young's experiment is (in standard notation) $d \sin\theta$. Define d and θ on the diagram. Hence, infer the standard expressions for the locations of bright and dark fringes in the intensity pattern on the screen. [10]

(c) i. Two speakers, one at $x = 0$ and one at $x = 20$ m, emit sound in phase with $\lambda = 7$ m. At what positions x between the two speakers will the interference be constructive? [5]

ii. A source of 500-nm light illuminates two slits separated by 0.5 mm. The first dark fringe on a distant screen is found 5 mm away from the central intensity peak. What is the distance from the slits to the screen? [5]