**BACHELOR IN COMPUTER APPLICATIONS**

**Term-End Examination**

**June, 2008**

**CS-601 : DIFFERENTIAL AND INTEGRAL CALCULUS WITH APPLICATIONS**

Time : 2 hours          Maximum Marks : 60

**Note :** Question number 1 is **compulsory**. Answer any **three** questions from the rest. Use of calculator is allowed.

1. (a) A function f(x) is defined as follows :

$$f(x) = \begin{cases} x+1 & x < 1 \\ x-1 & x > 1. \end{cases}$$

Discuss the continuity at $x = 1$.

(b) Evaluate

$$\lim_{x \to 0} \frac{x^2 - 2x}{\sin 3x}$$

(c) A balloon which remains spherical has a diameter $\frac{3}{2}(2x + 3)$. Determine the rate of change of volume w.r.t. x.

(d) Find the point on the curve

$$y = 2x^3 - 3x + 5$$

at which the tangent makes an angle of $45°$ with the positive direction of x-axis.

(e) If $\sin y = x \sin (a + y)$, prove that

$$\frac{dy}{dx} = \frac{\sin^2 (a + y)}{\sin a}.$$

(f) A particle is moving in a straight line according to the formula

$$x = t^3 - 9t^2 + 3t + 1,$$

where x is measured in metres and t in seconds. When the velocity is $-24$ m/s, find the acceleration.

(g) Can Rolle's theorem be applied to the following function ? Find 'C' in case it can be applied.

$$f(x) = x^3 - 2x \quad on \quad [0, 1]$$

(h) Evaluate

$$\int x^3 e^{x^4} \, dx$$

(i) By means of graph discuss the continuity of the following function :

$$f(x) = \begin{cases} 7 & x > 0 \\ 0 & x \leq 0 \end{cases}$$

(j) State which of the following are even functions and which are odd :

(i) $f(x) = 7x^2 - 11$

(ii) $f(x) = 2x^2 - 13x$

(iii) $f(x) = \tan x$

(iv) $y = \sec 2x$

(v) $f(x) = \sin^3 x$

(vi) $f(x) = e^{3x} - e^{-3x}$        $10 \times 3 = 30$

2. (a) Find the value of K for which the function

$$f(x) = \begin{cases} \dfrac{\sin 5x}{3x}, & x \neq 0 \\ K, & x = 0 \end{cases}$$

is continuous at   $x = 0$.

(b) Evaluate

$$\lim_{x \to 0} \frac{\sqrt{1 + x} - \sqrt{1 - x}}{x}$$

(c) Show

$$\frac{dy}{dx} = \frac{-1}{2(1 + x^2)} \quad \text{if} \quad y = \tan^{-1}(\sqrt{1 + x^2} - x)$$

       $3 + 3 + 4$

3. (a) Find the equation of the tangent and the normal to the curve

$$y = 5x^2 - 2x + 3$$

at $(1, 6)$.

(b) Evaluate

$$\int x^2 \cos x \; dx$$

(c) For the function

$$S = K \; x \; (4r^2 - x^2),$$

find the value of x for which S is maximum, K and r are constant. 3+3+4

4. (a) Evaluate

$$\int_0^{\pi/2} \sqrt{\sin x} \; \cos x \; dx$$

(b) Find the area bounded by

$$y^2 = 9x \quad \text{and} \quad x^2 = 9y.$$

(c) Find the perimeter of the cardioid

$$r = a \; (1 + \cos \theta) \qquad 3+3+4$$

5. (a) If $y = (\sin^{-1} x)^2$, prove that

$$(1 - x^2) \; y_2 - xy_1 - 2 = 0.$$

Using Leibnitz's theorem, show that

$$(1 - x^2) \; y_{n+2} - (2n + 1) \; xy_{n+1} - n^2 \; y_n = 0$$

(b) The velocity $v$ (km/min) of a moped which starts from rest, is given at fixed intervals of time t (min) as follows :

| t | 0 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 |
|---|---|----|----|----|----|----|----|----|----|----|----|
| $v$ | 0 | 10 | 18 | 25 | 29 | 32 | 20 | 11 | 5 | 2 | 0 |

Estimate approximately the distance covered in 20 minutes. 5+5