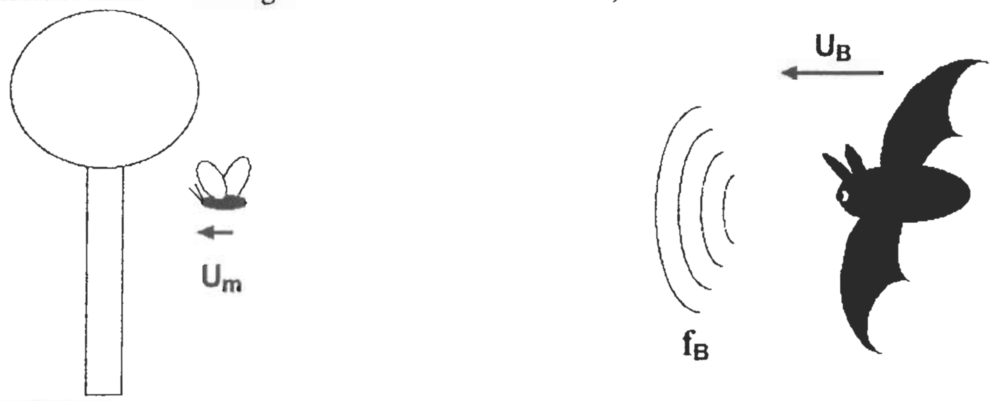
Vibrations & Waves Homework Sheet 4
QUESTIONS
(given out: Monday 14 February 2005)

Covers material in V & W Lectures 10, 11, 12

1) (This is the most complicated situation you can get for the Doppler effect - but does show clearly how you get approximately *twice* the frequency shift when a wave is bounced off something and returns to an observer)



A bat flying at speed $\mathbf{u_B}$ is out hunting for his supper. Using echo location he detects a moth flying at speed $\mathbf{u_m}$ away from him towards a tree. If the bat emits clicks of frequency $\mathbf{f_B}$, by considering the change in wavelength and period whenever the sound wave is emitted and reflected, show that the echoes received by the bat from the tree and moth have respective frequencies:

$$f_{tree} = \left(\frac{v + u_B}{v - u_B}\right) f_B \qquad f_{moth} = \frac{(v + u_B)(v - u_m)}{(v - u_B)(v + u_m)} f_B$$

where **v** is the speed of sound in air.
To maximise their sensitivity the ears of many bats have a high quality factor Q peaked at $\mathbf{f_B}$. Can you see a problem which arises from this given the situation above? To get round this problem some bats sweep the emission frequency during each echo-location "click". If $\mathbf{u_B}$ = 4 m/s, $\mathbf{u_m}$ = 0.1 m/s and **v** = 344 m/s, what range of frequencies would the above bat need to *emit* to detect the moth and tree at 10kHz? (ie. What $\mathbf{f_B}$ is needed to make $\mathbf{f_{tree}}$ and $\mathbf{f_{moth}}$ equal to 10kHz).

2) Show that:

$$v_g = v - \lambda \frac{dv}{d\lambda}$$

Using this formula or otherwise, find the group velocity of the following water waves:

(i) For ripples dominated by surface tension: $v = \sqrt{\sigma k / \rho}$

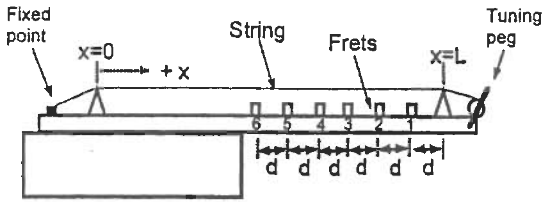
(ii) For surface gravity water waves in shallow water: $v = \sqrt{gh}$

(iii) For surface gravity water waves in deep water: $v = \sqrt{g / k}$

where **σ** is the surface tension, ρ is the density of water, **h** is the water depth, g the gravitational constant.
In each case, what type of dispersion is occurring?
Sketch a dispersion curve containing all three cases.

SMVWQP Exam May 2004 V&W Long Question (40 minutes)

7. A guitar string is fixed at one end. The other end it is attached to a tuning peg which, when rotated, allows the tension T^* in the string to be adjusted. The part of the string which vibrates is between two ridges at $x = 0$ and $x = L$. Other lower ridges, called frets, on the guitar neck allow the musician to adjust the length of the part of the string which vibrates by pushing the string down on to the fret with a finger. Let the frets numbered from $m = 1$ to 6 be equally spaced along the guitar neck at intervals of d.



(i) (a) By considering the addition of two travelling transverse plain waves going in opposite directions along the string:

$$y_+(x,t) = A\cos(\omega t - kx + \phi)$$

$$y_-(x,t) = -A\cos(\omega t + kx + \phi)$$

show that the guitar string will support standing waves of wavelength:

$$\lambda_n = 2\,(L - md)/n$$

where n is a positive integer and m is the number of the fret where the musicians finger is placed ($m = 0$ if the string is not held down on any fret). [5 marks]

(b) The phase speed of a transverse wave on a string is given by:

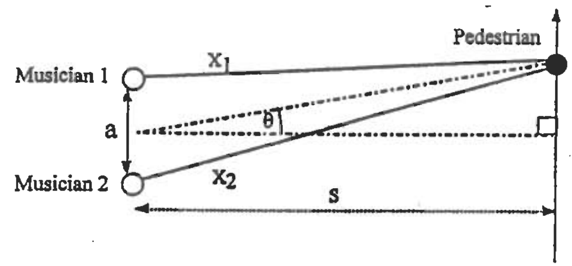
$$\upsilon = \sqrt{T*/\sigma}$$

where σ is the mass per unit length of the string. If the musician wishes the fundamental frequency ($n = 1$) to be equal to 1 kHz when no frets are used ($m = 0$), what tension T^* needs to be applied to the string? Take $\sigma = 10^{-3}\,\text{kgm}^{-1}$ and $L = 1$ m. [2 marks]

(c) A second identical string on the same guitar is adjusted to the same T^*. The musician then plays the first and second string simultaneously with his finger on no fret ($m = 0$) on the first string and fret $m = 2$ on the second string. This produces beats. What is their frequency ν_{beat} if $d = 0.05$ m. [2 marks]

Symbol note:
In this years course I have used:
v = phase velocity (Times lower case v) f = frequency (Times lower case f)
In the exam paper I have used:
υ = phase velocity (Ariel lower case v) ν = frequency (Greek nu)

(ii) (a) The musician goes to the local town square and tries to make some money by playing his guitar. A second musician turns up and stands a distance a away from the first. They both tune their guitars by playing the same note at 1 kHz. A pedestrian walks parallel to them along the other side of the square a distance s away. As he walks the sound intensity from the guitars varies with his position.



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Show that the sound intensity will reach a maximum when:

$$\sin\theta_{\max} = \frac{p\upsilon}{a\nu}$$

where p is an integer, υ is the speed of sound in air and ν is the wave frequency. [3 marks]

(b) Using this, calculate the distance Δy the pedestrian walks between maximum points of sound intensity. Let $s = 50$ m, $a = 5$ m and $\upsilon = 344\,\text{m}\,\text{s}^{-1}$. [3 marks]

(c) Each guitar emits an average power of 1 W. Assume that the sound wave moves out in a hemisphere (does not penetrate the ground). What is the maximum intensity the pedestrian will hear as he walks down the road? [3 marks]

[TOTAL 18 marks]