Vibrations & Waves Homework Sheet 3
QUESTIONS
(given out: Monday 7 February 2005)

Covers material in V & W Lectures 5, 6, 7, 8, 9

1) A mechanical forced oscillator system obeys the following equation of motion:

$$1\frac{d^2x}{dt^2} + 100\frac{dx}{dt} + 20{,}000x = 3\exp(j\omega t)$$

where all quantities are in SI units. Calculate (i) the frequency f of the driving force at which the maximum average power is transferred to the oscillator, (ii) the maximum average power P_{av} absorbed by the system at this frequency, (iii) the bandwidth Δf of the oscillator and (iv) the highest and lowest frequencies f_1 and f_2 over which useful power can be transferred to the oscillator.
If the driving force was suddenly switched off, how long would the oscillator take to come to rest if the driving force frequency f was (v) at the point of the maximum displacement of the oscillator and (vi) at the point of maximum power transfer. Assume that motion less than 0.1 mm cannot be detected.

2) Two identical simple pendulums of length L and mass m are placed next to each other. The masses are coupled together by a spring of force constant S. The distance between the two pendulums is such that when they are both at rest the spring exerts no force between the two masses. By writing down the equations of motion of both masses and combining them to give the normal co-ordinates, show that the angular frequencies of the two normal modes of the system are:

$$\omega_1 = \sqrt{g/L} \qquad\qquad \omega_2 = \sqrt{g/L + 2S/m}$$

Sketch the motion of the two normal modes.

3) Show that the following waves obey the wave equation:

(i) $\psi(x,t) = 0.2\exp(j(628t - 0.01x + 3.14))$

(ii) $\psi(x,t) = (0.1 - 0.15j)\exp(j(31.4t + 0.1x))$

(iii) $\psi(x,t) = 2\cos(31.4t - 0.7x)$

(iv) $\psi(x,t) = 3\sin(628t - 0.7x + 1)$

(v) $\psi(x,t) = 5\cos(100t - 30x)\sin(100t - 30x)$

and find the phase velocity **v**, period T and wavelength λ (all numbers are in SI units).

4) An underwater sonar emitter is lowered from a ship into the sea. The water has density ρ and bulk modulus $\boldsymbol{B}$. Consider an element of water of thickness Δx and area $\boldsymbol{a}$. For a liquid, changes of pressure in the $\boldsymbol{x}$ direction will result in a force in that direction of:

$$F = B\frac{\partial^2 X(x,t)}{\partial x^2} a\Delta x$$

where $\boldsymbol{X}$ is the coordinate governing the motion of molecules in the $\boldsymbol{x}$ direction. If ρ = 10^3 kg/m^3 and bulk modulus $\boldsymbol{B}$ = 2.18x10^9 N/m^2, what is the velocity of the sound waves in water?

The motion of the water molecules in the sound wave follows:

$$X(x,t) = X_0 \exp(j(\omega t - kx + \phi))$$

By considering the total energy of the motion of the water, show that the intensity of the wave is given by:

$$I = \frac{1}{2} B\omega k X_0^2$$

The sonar emits a pulse of average power 10 W at a frequency of 1 kHz. Assume that the sonar emitter is a sphere of radius 10 cm. What is the maximum displacement of a water molecule at the sonar emitter? What is the intensity of the sound waves at a distance of 100m from the sonar emitter?