Vibrations & Waves Classwork 4
(Monday 14 February 2005)

A physics Lecturer is in a reality game-show called *"I'm an Academic – Get me out of here!"* in the Australian jungle. Unless he can solve the following physics problems he will have to eat some large grubs and beetles…..

He sits at one end of a large pool of water. 10m away exactly opposite him two bamboo pipes are extended over the other end of the pool. The pipes are separated by 1m. Water drips continuously out of the end of each pipe. The drips create a series of circular wave-fronts moving out continuously across the pond from beneath the end of each pipe. These have wavelength λ = 0.2m.

(a)i) The circular ripples created by the two dripping pipes undergo interference with each other. Let θ be an angle measured from a point exactly between the two pipe ends and from a line perpendicular to them (so that they lie at $\theta = \pm\pi/2$). Show that constructive interference between the ripples occurs when:

$$\theta \approx m\lambda/a$$

where *a* is the separation between the ends of the two pipes (1m) and *m* is an integer.
ii) The physicist sits exactly opposite the two pipes at an angle $\theta = 0$ so that the ripples reaching him have a maximum amplitude. At what distance to his left and right do the ripples also have a maximum amplitude?

(b)i) A rectangular wooden box floats in front of the physicist of length *d*. The box is filled with some water, but not enough to make it sink. Due to the motion of the box in response to the ripples, a standing wave forms in the box. This has antinodes at each end of the box. What possible wavelengths can a standing wave have in the box? If the standing wave has the same frequency as the ripples, what mode is present if *d* = 0.7 m.
ii) A circular wooden bowl also floats in front of the physicist. It has diameter *d*. The bowl is also filled with water, again not enough to make it sink. What possible wavelengths can a circularly symmetric standing wave have in the bowl? Could the mode with the same wavelength as the rectangular box occur? [Think of circular symmetry and what must happen in the centre of the bowl].

The waves in the pond have a phase velocity given by:

$$v = \sqrt{g/k}$$

Take g = 10 ms^{-2}.

c) Suddenly a small crocodile jumps into the water by the pipes 10m away from the physicist. This creates a wave pulse containing a distribution of wavelengths from 0.1m to 0.4m and peaked at 0.25m.
(i) What is the initial velocity of the pulse?
(ii) What time will it take for the shortest and longest wavelength components to reach the physicist?
(iii) When the 0.4m waves arrive at the physicist, how wide will the pulse be? If the pulse was initially 0.3 m wide, do you think it still exists?