<u>Vibrations & Waves Classwork 2 – (Solutions)</u>
(Problems given out: Monday 24 January 2005)

M = 95 kg, P_{av} = 400 W, ω_{max} = 83.8 rad/s, m = 5 kg.
x_0 = 0.01m, R = 0.3m,

(i) $F_0 = \frac{mv^2}{R} = \frac{m}{R}\left(\frac{2\pi R}{T}\right)^2 = m\omega^2 R$

At ω_0 => $F_0 = m\omega_0^2 R$

(ii)

$$Z_m = \sqrt{r^2 + (M\omega - s/\omega)^2}$$
$$= \sqrt{r^2 + (M\omega_0 - s/\omega_0)^2}$$
$$= \sqrt{r^2 + (M\sqrt{s/M} - s/\sqrt{s/M})^2}$$
$$= \sqrt{r^2 + (\sqrt{sM} - \sqrt{sM})^2} = r$$

(iii) $x_0 = \frac{F_0}{\omega Z_m}$ => $x_0 = \frac{F_0}{\omega_0 r}$

(iv) $P_{av} = \frac{rF_0^2}{2Z_m^2}$ => $P_{av} = \frac{F_0^2}{2r}$

(v)

$$r = \frac{F_0}{\omega_0 x_0} = \frac{F_0^2}{2P_{av}} \Rightarrow \frac{1}{\omega_0 x_0} = \frac{F_0}{2P_{av}}$$

$$\Rightarrow \frac{1}{\omega_0 x_0} = \frac{m\omega_0^2 R}{2P_{av}} \Rightarrow \omega_0^3 = \frac{2P_{av}}{mRx_0}$$

(vi) $\omega_0 = \sqrt[3]{\frac{2P_{av}}{mRx_0}} = \sqrt[3]{\frac{800}{5 \times 0.3 \times 0.01}} = 37.64 rad/s$

(vii) $F_0 = 5 \times 37.6^2 \times 0.3 = 2{,}120N$

$$r = \frac{F_0}{\omega_0 x_0} = \frac{2120}{37.64 \times 0.01} = 5{,}630 Ns/m$$

$$s = \omega_0^2 M = (37.64)^2 95 = 134{,}600 N/m$$

(viii)

[NOTE: I have removed the calculation of ω_r from the original Question and Answer sheet – it did'nt work! The system is so close to critical (see (ix) below) that $\left(\omega_0^2 - \frac{r^2}{2M^2}\right) < 0$ but $\left(\omega_0^2 - \frac{r^2}{4M^2}\right) > 0$. Hence, is ω_r meaningless in this case. We work out ω_r from when the amplitude x_0 of forced SHM is a maximum. However, close to critical the peak in the curve disappears! Hence you can't calculate a value of ω_r].

$$Q = \frac{M\omega_0}{R} = \frac{95 \times 37.64}{5630} = 0.635$$

$$\Delta\omega = \frac{\omega_0}{Q} = \frac{37.64}{0.635} = 59.3 rad/s$$

$$\omega_0 + \frac{\Delta\omega}{2} = 37.64 + \frac{59.3}{2} = 67.3 rad/s$$

=> ω_{max} = 83.8 rad/s => outside bandwidth

(ix)

$$\frac{r^2}{4M^2} = \frac{5630^2}{4 \times 95^2} = 878$$

$$\omega_0^2 = 37.64^2 = 1417$$

$$\Rightarrow \omega_0^2 > \frac{r^2}{4M^2}$$

=> just lightly damped - but very close to critical => well designed washing machine