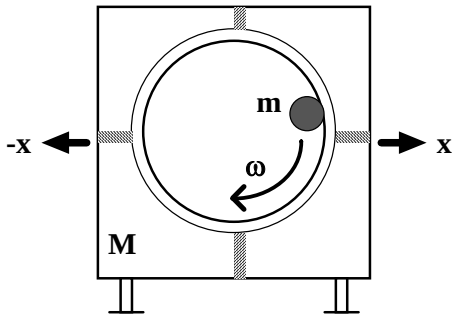
Vibrations & Waves Classwork 2
(Monday 24 January 2005)

My washing machine undergoes violent horizontal oscillations as the drum accelerates towards full spin-speed. Why does this imply that its a *really* bad idea to put lots of towels in the same wash……..

Dr Campbell's washing machine



Treat the main body of the machine on four metal/rubber supports as a damped simple harmonic oscillator of mass M, spring constant s and mechanical resistance r. When displaced from equilibrium by a horizontal force it will tilt sideways, but will return to equilibrium. Suspended in the centre of the machine is the drum. When filled with washing, the washing tends to gather at the bottom of the drum. When the drum rotates rapidly the washing will therefore move in a circle. This will provide a periodic force $F_0\cos(\omega t)$ to the main body of the machine. The whole system therefore acts as a forced simple harmonic oscillator. By consulting the manual for my washing machine I know M = 95 kg, the electric motor power is 0.4 kW, the maximum spin-speed is 800 rpm, and the maximum dry clothes capacity m = 5kg.

(i) Uniform circular motion of mass m at velocity v in a path with radius R requires a radial force towards the centre of mv^2/R. Write down an equation relating F_0 to m, ω and R, where R is the radius of the drum. Hence write down an equation for F_0 at ω_0.
(ii) When $\omega = \omega_0$, what is the general equation for the magnitude of the mechanical impedance Z_m?
(iii) What general equation gives the average power P_{av} in forced SHM at ω_0?
(iv) What general equation gives the amplitude x_0 in forced SHM at ω_0?
(v) By combining the above equations show that:

$$\omega_0^3 = \frac{2P_{av}}{mRx_0}$$

(vi) I estimate an amplitude x_0 = 1 cm when my machine passes through resonance as the drum accelerates to spin. Also assume P_{av} = 400W [probably an overestimate – drive power also lost in other ways than oscillating the machine]. Taking R = 30 cm and m = 5 kg, calculate ω_0.
(vii) From (i) calculate F_0 at ω_0. Then from (iv) calculate r. Also from M = 95kg calculate a value of s from ω_0.
(viii) Calculate Q and the bandwidth $\Delta\omega$. Does the maximum spin-speed angular frequency lie within the bandwidth?
(ix) Is my washing machine lightly, critically or heavily damped?