Vibrations & Waves Classwork 1 – (Solutions)

(Problems given out: Monday 17 January 2005)

i) Lightly damped SHM.

Take minibus + students => m
Suspension => stiff spring => r & s

ii) $$x(t) = A\exp(-\frac{r}{2m}t)\cos(\omega' t + \phi)$$

$$\omega' = \sqrt{\left(\frac{s}{m} - \frac{r^2}{4m^2}\right)} = \sqrt{\left(\omega_0^2 - \frac{r^2}{4m^2}\right)}$$

iii) For long decay: ω' ≈ ω_0

$$\omega_0^2 = \frac{s}{m}$$

We know m = (70x20)+1000 = 2400 kg
Therefore: s = 94,700 N/m

iv) Initial amplitude at t=0 => 0.05m
After 10s => 0.001m

Use: $A(t) = A(t=0)\exp(-\frac{r}{2m}t)$

=> $$r = -\frac{2m}{t} Ln\left|\frac{A(t)}{A(t=0)}\right|$$

Therefore: r = 1,880 Ns/m

v)

$$\omega' = \sqrt{\left(\omega_0^2 - \frac{r^2}{4m^2}\right)} \Rightarrow \omega_0^2 = \left(\frac{r^2}{4m^2}\right) + (\omega')^2 = \frac{s}{m}$$

s = 95,100 N/m
ω_0 = 6.295 rad/s
variation in period = 0.19%

vi) $$\boldsymbol{TE(t) = \frac{1}{2}sA^2\exp(-\frac{r}{m}t)}$$

$$\boldsymbol{TE(t) = \frac{1}{2}95{,}020(0.05)^2\exp(-\frac{1880}{2400}t)}$$

$$\boldsymbol{= 119\exp(-0.783t)}$$

t = 0 => TE = 119 J
t = 10sec => TE = 0.0471 J

vii) $$Q = \frac{m\omega'}{r} = \frac{2400.2\pi}{1880} = 8.03$$

viii)
Initial amplitude at t=0 => 0.05m
After 1s => 0.0001m
=> $$r = -\frac{2m}{t} Ln\left[\frac{A(t)}{A(t=0)}\right]$$
Therefore: r = 29,830 Ns/m

ix)
$$\frac{s}{m} = \left(\frac{r^2}{4m^2}\right) + (\omega')^2$$
s = 187,400 N/m

x) $$Q = \frac{2400.2\pi}{29830} = 0.506$$

xi) Want critical damping:

$$\frac{s}{m} = \frac{r^2}{4m^2} \Rightarrow s = \frac{r^2}{4m} \Rightarrow m = \frac{r^2}{4s}$$

m = 1187 kg
1187 - 1000 = 187 kg = about 2 to 3 students
=> about 17 to 18 jump out