Problem Sheet 1
Lectures 0–4

Learning Outcomes

Jargon
Macroscopic, microscopic, absolute temperature, mole, latent heat, conduction, convection, radiation, internal energy, degrees of freedom, equilibrium, quasistatic process, isothermal, adiabatic, heat capacity, constant volume and constant pressure heat capacities, specific heat, molar specific heat.

Notation
K (kelvin), k_B, R, N_A, U, ΔU, ΔQ, ΔW, C_V, C_P, c_V, c_P, C_{V_m}, C_{P_m}, γ (ratio of specific heats).

Concepts
Qualitative differences between solids, liquids and gases at microscopic level; thermal motion; heat flow; ideal gas equation of state (and assumptions needed to derive it); relationship between P and U in ideal gas; theorem of equipartition of energy; qualitative distinction between work and heat at the microscopic level; first law of thermodynamics; calculating work done in quasistatic compression/expansion; plotting quasistatic process on PV diagram; process dependent nature of work and heat; relationship between heat capacities and number of degrees of freedom; relationship between P and V in adiabatic process.

Problems

1. (a) An ideal gas has a pressure of 1 atmosphere, a volume of 0.5 m^3 and a temperature of 300 K. Calculate the number of molecules it contains.
   [1 atmosphere $= 1.0 \times 10^5$ N m^{-2}.]

   (b) Three moles of an ideal gas is at a pressure of 10^2 N m^{-2} and has a temperature of -10° C. Calculate its volume.

   (c) Calculate the volume occupied by 1 mole of an ideal gas at standard temperature and pressure (STP), i.e. 273.15 K and 1.01×10^5 Pa (a useful number to remember!)

2. The plasma in a fusion reactor can be thought of as a mixture of two gases, an ion gas and an electron gas, both of which have a number density of 5.0×10^{19} particles m^{-3} and a temperature of 10^8 K. The volume of the plasma is 10^3 m^3. Assuming that both ion and electron gases can be treated as ideal gases with three degrees of freedom, calculate the pressure and internal energy of each gas separately. Hence find the pressure and internal energy of the whole plasma. Express the plasma pressure in atmospheres.

3. A monatomic ideal gas, initially at a pressure of 1 atmosphere, a volume of 0.5 m^3, and a temperature of 300 K, goes through the following four part quasistatic cycle:

   (1) increase of pressure at constant volume to 1.5 atmospheres.

   (2) expansion at constant pressure to 1 m^3,

   (3) reduction of pressure at constant volume to 1 atmosphere,

   (4) compression at constant pressure to 0.5 m^3,

   (a) Plot these four parts on a single PV diagram.
   (b) Calculate ΔT, the temperature change, for each part separately.
   (c) Calculate ΔU, the internal energy change, for each part separately.
   (d) Calculate the total internal energy change for the whole cycle.
   (e) Calculate ΔW, the work done on the gas, during each part separately.
   (f) Calculate the total work done *by* the gas over the whole cycle.
   (g) You should have found that after going through the whole cycle the internal energy of the gas is unchanged, but it has done a finite amount of work. Where has the energy for this come from?
   (h) How is the total work done by the gas indicated on the PV diagram of part (a)?

4. (a) Argon is a monatomic gas with an atomic mass of 40.0. A volume of 1 m^3 of Argon is at a temperature of 300 K, and a pressure of 10^5 N m^{-2}. Assuming it can be treated as an ideal gas, calculate the number of molecules, the number of moles, and the mass of the gas.
   (b) Calulate the constant volume and constant pressure heat capacities, C_V and C_P (in J K^{-1}), and their ratio C_P/C_V.
   (c) Calulate the constant volume and constant pressure specific heats, c_V and c_P (in J K^{-1} kg^{-1}), and their ratio c_P/c_V.
   (d) Calulate the constant volume and constant pressure molar specific heats, denoted C_{V_m} and C_{P_m}, (in J K^{-1} mol^{-1}), and their ratio C_{P_m}/C_{V_m}.

5. (a) Consider two blocks of iron, one of mass 5 kg and initial temperature 50°C, the second of mass 10 kg and initial temperature 0°C. They are placed in thermal contact and allowed to reach equilibrium at the same temperature. Assuming there are no thermal losses to the surroundings, calculate the final temperature.
   (b) Each breath a student takes in a room at temperature of 20° is heated to a temperature of 37° before being expelled. If an average lung is 0.50 litres and the specific heat of air is 1020 J/kg K ($\rho_{air} = 1.3 \times 10^{-3}$ kg/litre), what is the heat lost over an hour due to breathing if the respiration rate is 20 breaths per minute.
   (c) The same student has a skin temperature of 35° and an emissivity of close to 1. Calculate the heat flow by radiation assuming he is well covered except for his head which has a surface area of $5 \times 10^{-2}\,\mathrm{m}^2$. What is the heat flow due to radiation per hour?
   (d) Due to surface evaporation, convection and conduction, the heat loss per student is closer to 50 W. If 100 such students were to release this much energy in an lecture, how much would the air temperature change in one hour if the room has a volume 4000 m^3 if there was no air conditioning?

6. (a) An adiabatic process is one in which there is no heat flow ($\mathrm{d}Q = 0$). Recalling that the internal energy of an ideal gas is $U = \frac{n_d}{2} N k_B T$ (where n_d id the number of degrees of freedom), show that the first law of thermodynamics for an adiabatic process in an ideal gas can be written: $\frac{\mathrm{d}T}{T} = -\frac{2}{n_d}\frac{\mathrm{d}V}{V}$.

   (b) Integrate this to show that such a process satisfies: $TV^{2/n_d} =$ constant.

   (c) Show that the equation found in part (b) can be rewritten as $PV^\gamma =$ constant, [different constant from part (b)] where $\gamma = C_P/C_V$ i.e., γ is the ratio of heat capacities; usually known as the ratio of specific heats which, of course, is the same thing (see Q. 4). What is the value of γ for a monatomic gas?

   (d) A monatomic ideal gas initially has pressure P_0 and volume V_0. It then undergoes isothermal expansion to volume $2V_0$. A second monatomic ideal gas (with the same value of N) also initially has pressure P_0 and volume V_0. it undergoes an adiabatic expansion to $2V_0$. Sketch both of these processes on the same PV diagram. Which curve is steeper? Which ends at the higher temperature?

7. (a) Use the first law of thermodynamics to show that the work done in an adiabatic process ($\mathrm{d}Q = 0$) in an ideal gas is given by

$$\Delta W = C_V (T_1 - T_0)$$

   where T_0 and T_1 are the initial and final temperatures.

   (b) Alternatively, the work done can be found by integrating the pressure with respect to volume. Do this for an adiabatic process in an ideal gas, recalling that $PV^\gamma =$ constant in such a process (previous question) to show that

$$\Delta W = \frac{1}{\gamma - 1} (P_1 V_1 - P_0 V_0)$$

   where P_0 and V_0 are the initial pressure and volume, and P_1 and V_1 are the final pressure and volume.

   (c) Show that the expressions obtained in parts (a) and (b) agree.

8. (a) The speed of sound in a gas is given by $v_s = \sqrt{\frac{\gamma P}{\rho}}$ where γ is the ratio of specific heats in the adiabatic law [Q. 6 (c)] and ρ is the density (in kg m^{-3}) of the gas. Show that if the sound speed is measured to be v_s at some temperature T then the number of degrees of freedom of the gas molecules can be found from:

$$n_d = 2\left(\frac{m v_s^2}{k_B T} - 1\right)^{-1}$$

   where m is the mass of a gas molecule.

   (b) The average mass per molecule in air is 4.82×10^{-26} kg. The speed of sound in air at 20°C is 344 m s^{-1}. Calculate n_d in air (round to the nearerst integer).

   (c) Is the answer to part (b) what you would have expected?

Numerical answers overleaf

Numerical Answers

1. (a) 1.22×10^{25} molecules

   (b) 65.6 m^3

   (c) 0.0224 m^3

2. $P_{ion} = P_{electron} = 6.9 \times 10^4$ Pa

   $U_{ion} = U_{electron} = 1.03 \times 10^8$ J

   $P_{total} = 1.37$ atmospheres

   $U_{total} = 2.07 \times 10^8$ J

3. (b) 150 K, 450 K, -300 K, -300 K

   (c) 3.79×10^4 J, 1.136×10^5 J, -7.64×10^4 J, -7.64×10^4 J

   (e) 0, -7.57×10^4 J, 0, 5.05×10^4 J

   (f) 2.52×10^4 J

4. (a) 2.42×10^{25} molecules, 40.1 moles, 1.60 kg

   (b) 500 JK^{-1}, 833 JK^{-1}, 1.67

   (c) 312 JK^{-1}kg^{-1}, 521 JK^{-1}kg^{-1}, 1.67

   (d) 12.5 JK^{-1}mol^{-1}, 20.8 JK^{-1}mol^{-1}, 1.67

5. (a) 16.7°C

   (b) 13,500 J

   (c) 13.7 J

   (d) 3.4°C

6. (c) $\gamma = 5/3$ for a monatomic ideal gas

8. (b) 5