SoM: Problem sheet 4, Answers

1. Consider iceberg of vol V.

Upward force on iceberg (buoyancy) = weight of iceberg

But buoyancy = weight of water displaced

∴ mass of water displaced = mass of iceberg = $920 \times V$ kg.

∴ vol of water displaced $= \frac{920 \times V}{1025} = 0.898V$

But this is the vol of the submerged part of the iceberg i.e. 89.8% is submerged.

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2 (a) speed from nozzle (u) given by

$\pi \times (0.4)^2 \times u = \pi \times (0.8)^2 \times 1.2$

$\rightarrow u = 4.8\,ms^{-1}$

(b) Vol flow rate $= \pi \times (0.8)^2 \times 1.2 = 2.41\ m^3 s^{-1}$

Time to fill tank $= \frac{20}{2.41} = 8.29s$

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3 (a) $\frac{4\pi r^3}{3} = b \rightarrow r = 2.49 \times 10^{-10}\,m$

(b) $T_c = \frac{8a}{27k_B b} = 128\,K$ (only 2K out!)

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4 (a) $U = 3Nk_BT$ ∴ dT produces $dU = 3Nk_BdT$

But at const vol $dQ = dU$ & $dQ = C_V dT$

$\rightarrow C_V = 3Nk_B$

(b) c_V = molar spec ht. $= 3N_Ak_B = 3R = 24.9\ \mathrm{J\,K^{-1}\,mol^{-1}}$

5: $\Delta L = 1.17\times10^{-5}\times 320\times(-30) = -0.112\,\mathrm{m}$

i.e. $\sim 11\,\mathrm{cm}$

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6 Consider cube of side L_0, vol $V_0 = L_0^3$

$V_{fin} = (L_0 + \Delta L)^3 = L_0^3\left(1 + \frac{\Delta L}{L_0}\right)^3$

$\simeq L_0^3\left(1 + 3\frac{\Delta L}{L_0}\right)$ (ignoring higher order terms)

$\Delta V = \frac{L_0^3\, 3\, \Delta L}{L_0}$ ← $\Delta L = \alpha L_0 \Delta T$

$= (3\alpha)(L_0^3)\,\Delta T$

(3α ↑ β; L_0^3 ↑ V_0)

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7 (a) $\frac{5}{2}k_BT = 7.18\times10^{-19} \rightarrow T = 2.08\times10^4\,\mathrm{K}$

(b) $\frac{3}{2}k_BT = 2.18\times10^{-18} \rightarrow T = 1.05\times10^5\,\mathrm{K}$

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8. (a) $U_{\mathcal{E}\,max} = \frac{1}{2}\epsilon_0 \mathcal{E}_{max}^2 = \frac{1}{2}\epsilon_0 \frac{n_0^2 e^2 d^2}{\epsilon_0}$

$$= \frac{n_0^2 e^2 d^2}{2\epsilon_0}$$

(b) $\frac{n_0^2 e^2 d_{max}^2}{2\epsilon_0} = \frac{n_0 k_B T}{2}$

$$\rightarrow d_{max}^2 = \frac{\epsilon_0 k_B T}{n_0 e^2} \quad (= \lambda_D^2)$$

(c) $\lambda_D = 6.91 \times 10^{-5}$ m (i.e. much smaller than a fusion plasma in a tokamak, which is several metres across).