Structure of Matter — M. Coppins
First Year Physics — 20.05.05

Problem Sheet 4
Lectures 9–11

Learning Outcomes

Jargon

Buoyancy, mass flow rate, isobar, isochor, critical point, triple point, melting, vaporization, sublimation, metallic bond, linear and volume thermal expansion coefficients, plasma.

Concepts

Pressure gradient force in fluid; Archimedes' Principle; continuity equation; PV and PT phase diagrams, showing boundaries between various phases; deriving an expression for the critical temperature using the van der Waals equation of state; classical expressions for the internal energy and constant volume heat capacity of a solid.

Problems

1. An iceberg of density 920 kg m^{-3} floats in seawater of density 1025 kg m^{-3}. What fraction of its volume is submerged?

2. (a) Water flows at 1.2 m s^{-1} through a hose-pipe of radius 0.8 cm. Calculate the speed with which it emerges from a nozzle of radius 0.4 cm.

   (b) How long would it take to fill a tank of volume 20 m^3 with this hose-pipe?

3. The van der Waals constants for Nitrogen are $a = 3.86 \times 10^{-49}$ J m^3 and $b = 6.49 \times 10^{-29}$ m^3.

   (a) Assuming that a Nitrogen molecule is spherical, estimate its radius.

   (b) Use the result of Q8, Problem Sheet 3, to estimate the critical temperature of Nitrogen. (For comparison the actual value is 126 K.)

4. (a) Assuming the classical result for the internal energy of a solid (Sec 11.1, Lec 11) show that the constant volume heat capacity of a solid is $C_v = 3Nk_B$. (This result is called the Dulong and Petit Law.)

   (b) Calculate the classical value of the molar specific heat of a solid in J K^{-1}.

5. A solid expands when its temperature is raised. A temperature increase of ΔT will produce an increase in the length of an object of $\Delta L = \alpha L_0 \Delta T$, where L_0 is the initial length and α is the *linear thermal expansion coefficient*.

   The Eiffel Tower is made of steel (linear thermal expansion coefficient $\alpha = 1.17 \times 10^{-5}$ K^{-1}). At 20^o C it has a height of 320 m. Calculate how much shorter it is at -10^o C.

6. The *volume thermal expansion coefficient*, β, is defined such that a temperature increase ΔT produces a volume increase $\Delta V = \beta V_0 \Delta T$, where V_0 is the initial volume. Show that for small temperature changes $\beta = 3\alpha$ (where α is defined in Q 5).

7. (a) The molecular dissociation energy of an H_2 molecule (i.e., the energy required to break the bond between the two atoms) is 7.18×10^{-19} J. Calculate the temperature at which the average molecular energy is equal to this energy. Assume the molecule has five degrees of freedom.

   (b) The ionization energy of Hydrogen is 2.18×10^{-18} J. Calculate the temperature at which the average atomic energy is equal to this energy.

8. Consider an exactly neutral Hydrogen plasma in which $n_e = n_i = n_0$ (n_e and n_i are the electron and ion number densities) and the charge density $\rho_q = (n_i - n_e)e$ is zero everywhere. If all the electrons in a layer of width d are shifted distance d in one direction a charged layer will be formed in the plasma. This sets up an electric field, the maximum value of which is $E_{max} = \dfrac{n_0 e d}{\epsilon_0}$ (don't bother to prove this unless you really want to).

   (a) The work done to move the electrons is stored in the electric field. Given that the energy density in an electric field is $u_E = \dfrac{1}{2}\epsilon_0 E^2$ (in J m^{-3}), write down an expression for the maximum value of u_E.

   (b) Small non-neutral regions can arise spontaneously in the plasma as a result of thermal fluctuations. The energy to form them comes from the translational kinetic energy of the electrons. The average energy per unit volume available to form the charged layer described above is $n_0 \times \dfrac{1}{2} k_B T$ (the relevant electron motion corresponds to one degree of freedom). Equating this to the maximum u_E gives an expression for d_{max}, the maximum width of the layer. Show that $d_{max} = \left(\dfrac{\epsilon_0 k_B T}{n_0 e^2}\right)^{1/2}$ (= the Debye length).

   (c) Calculate the Debye length in a magnetically confined fusion plasma in which $n_0 = 10^{20}$ m^{-3} and $T = 10^8$ K.

Numerical Answers

1. 89.8%
2. (a) 4.8 ms^{-1}, (b) 8.29 s
3. (a) 2.49×10^{-10} m, (b) 128 K.
4. (b) 24.9 JK^{-1}mol^{-1}.
5. 11.2 cm.
7. (a) 2.08×10^4 K, (b) 1.05×10^5 K.
8. (c) 6.91×10^{-5} m.