SoM : Problem Sheet 1, Answers

1(a) $N = \frac{PV}{k_BT} = \frac{1.01\times10^{5}\times 0.5}{1.38\times10^{-23}\times 300} = 1.22\times10^{25}$

(b) $V = \frac{Nk_BT}{P} = \frac{3N_Ak_BT}{P} = \frac{3RT}{P} = \frac{3\times 8.31\times 263}{10^2} = 65.6\,m^3$

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2 $P = Nk_BT/V = nk_BT$ $\quad$ n = no. density.

$P_{ion} = 5\times10^{19}\times 1.38\times10^{-23}\times10^{8} = 6.9\times10^{4}$ Pa

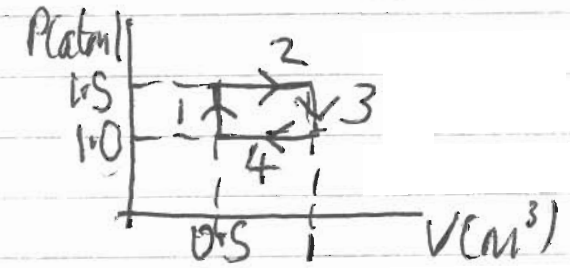
$P_{elec} = P_{ion}$ (same n & T)

$U_{ion} = \frac{3}{2}PV = 1.03\times10^{8}\,J = U_{elec}$

$P_{tot} = P_{ion}+P_{elec} = 1.38\times10^{5}$ Pa $= 1.37$ atmospheres

$U_{tot} = U_{ion}+U_{elec} = 2.07\times10^{8}\,J$

3(a)



(b) (1) $V = \text{const} \rightarrow T/P = \text{const} \rightarrow T_{fin} = 1.5\times 300 = 450\,K \rightarrow \Delta T_1 = 150\,K$

(2) $P = \text{const} \rightarrow T/V = \text{const} \rightarrow T_{fin} = 2\times 450 = 900\,K \rightarrow \Delta T_2 = 450\,K$

(3) $V = \text{const} \rightarrow T_{fin} = 900/1.5 = 600\,K \rightarrow \Delta T_3 = -300\,K$

(4) $P = \text{const} \rightarrow T_{fin} = 600/2 = 300\,K \rightarrow \Delta T_4 = -300\,K$

(c) $U = 3/2\, Nk_BT$, $N = 1.22\times10^{25}$ molecules (see Q 1(a))

$\therefore \Delta U = \underbrace{\frac{3}{2}Nk_B}_{252.5\,JK^{-1}}\Delta T$

(1) $\Delta U_1 = 252.5\times 150 = 3.79\times10^{4}\,J$

(2) $\Delta U_2 = 252.5\times 450 = 1.136\times10^{5}\,J$

(3) $\Delta U_3 = 252.5\times(-300) = -7.66\times10^{4}\,J$

(4) $\Delta U_4 = -7.66\times10^{4}\,J$

[alternatively calculate U from $3/2\,PV$]

(d) $\Delta U_{tot} = 0$ $(\Delta T_{tot} = 0)$

(e) (1) $W_1 = 0$ $(dV = 0)$

(2) $W_2 = -\int_{0.5}^{1} P\,dV$ & $P = 1.5\times1.01\times10^5\ \mathrm{Nm^{-2}}$

$= -1.5\times1.01\times10^5\times(1-0.5) = -7.57\times10^4\ \mathrm{J}.$

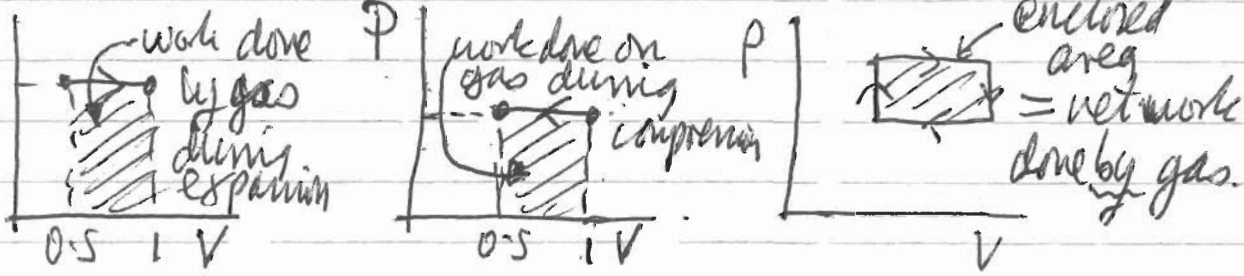
(3) $W_3 = 0$ $(dV = 0)$

(4) $W_4 = -\int_{1}^{0.5} P\,dV = +5.05\times10^4\ \mathrm{J}.$

(f) Work done BY gas $= +7.57\times10^4 - 5.05\times10^4 = 2.52\times10^4\ \mathrm{J}$

(g) At each stage energy enters or leaves gas as heat as well as work. 1st Law: $\Delta U_{tot} = W_{tot} + Q_{tot}$. But $\Delta U_{tot} = 0$
$\therefore Q_{tot} = -W_{tot}$ = tot work done by gas.

(h)



4(a) $N = \dfrac{pV}{k_BT} = \dfrac{10^5\times1}{1.38\times10^{-23}\times300} = 2.42\times10^{25}$ molecules

$n_{moles} = N/N_A = 40.1$, $M_{gas} = N\times40.0\times1.66\times10^{-27} = 1.60\ \mathrm{kg}$

(b) Monatomic $\rightarrow n_d = 3$, $C_V = (n_d/2)Nk_B = 500\ \mathrm{JK^{-1}}$

$C_p = C_V + Nk_B = 833\ \mathrm{JK^{-1}}$, $C_p/C_V = 1.67$

(c) $c_V = C_V/M_{gas} = 312\ \mathrm{JK^{-1}kg^{-1}}$, $c_p = C_p/M_{gas} = 521\ \mathrm{JK^{-1}kg^{-1}}$

$c_p/c_V = 1.67$

(d) $c_V = C_V/n_{moles} = 12.5\ \mathrm{JK^{-1}mol^{-1}}$, $c_p = C_p/n_{moles} = 20.8\ \mathrm{JK^{-1}mol^{-1}}$

$c_p/c_V = 1.67$

[All 3 ratios are $(n_d+2)/n_d$; see Eq. 3.3.4]

5 Block A: $T_{init} = 323\,K$, $m_A = 5kg$
Block B: $T_{init} = 273\,K$, $m_B = 10kg$.
Both have same final temp T_f.
Heat out of A = heat into B.
$C^A(323 - T_f) = C^B(T_f - 273)$
ht cap's
$C^B/C^A = m_B/m_A = 2$

$\therefore 323 - T_f = 2(T_f - 273) \to T_f = (323+546)/3 = 289.7K\ (=16.7°C)$

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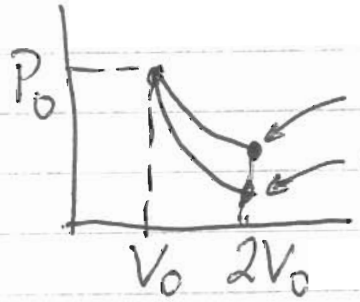
6 (a) $dQ = 0 \to dU = -PdV = -(Nk_BT/V)\,dV$
But $dU = \frac{n_d}{2}Nk_BT \;\therefore\; \frac{n_d}{2}Nk_BT = -\frac{Nk_BT}{V}dV \to \frac{dT}{T} = -\frac{2}{n_d}\frac{dV}{V}$

(b) Integrate: $\ln T = c_1 - \frac{2}{n_d}\ln V = c_1 - \ln V^{2/n_d}$
int const
$\therefore \ln(TV^{2/n_d}) = c_1 \to TV^{2/n_d} = c_2$ ← e^{c_1}

(c) $T = \frac{PV}{Nk_B} \to \frac{PV^{(1+2/n_d)}}{Nk_B} = c_2 \;\;\therefore\; PV^\gamma = c_3$ ← Nk_Bc_2
$\gamma = (n_d+2)/n_d = C_P/C_V$ (see eq 3.3.5).
$\gamma = 5/3$ for monatomic ideal gas

(d)



isotherm: $P \propto V^{-1}$ (ends at higher T)
adiabat: $P \propto V^{-5/3}$ (steeper)

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7 (a) IG: $U = C_VT \to dU = C_VdT$. Adiabatic $dW = dU = C_VdT$
$W = \int_{T_0}^{T_1} C_V\,dT = C_V(T_1 - T_0)$

(b) $W = -\int_{V_0}^{V_1} PdV = -\int \frac{c}{V^\gamma}dV$ $c = \text{const} = P_0V_0^\gamma = P_1V_1^\gamma$

$$= -C\left[\frac{V^{1-\gamma}}{1-\gamma}\right]_{V_0}^{V_1} = \frac{C}{\gamma-1}\left[V_1^{1-\gamma} - V_0^{1-\gamma}\right]$$

But $P_0V_0^{\gamma} = C \rightarrow V_0^{-\gamma} = P_0/C \rightarrow V_0^{1-\gamma} = P_0V_0/C$

Similarly $V_1^{1-\gamma} = P_1V_1/C$

$$\therefore W = \frac{1}{\gamma-1}\left[P_1V_1 - P_0V_0\right]$$

(c) $\gamma = \frac{C_P}{C_V} = \frac{C_V + Nk_B}{C_V} = 1 + \frac{Nk_B}{C_V} \quad \therefore \frac{1}{\gamma-1} = \frac{C_V}{Nk_B}$

from (b) $W = \frac{C_V}{Nk_B}\left[\underset{Nk_BT_1}{\underset{\uparrow}{P_1V_1}} - \underset{Nk_BT_0}{\underset{\uparrow}{P_0V_0}}\right] = C_V(T_1 - T_0)$

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8 (a) $V_S^2 = \frac{\gamma P}{\rho} = \frac{\gamma Nk_BT/V}{mN/V} = \frac{\gamma k_BT}{m}$

But $\gamma = \frac{n_d+2}{n_d} = 1 + \frac{2}{n_d} \quad \therefore \frac{2}{n_d} = \frac{mV_S^2}{k_BT} - 1 \rightarrow n_d = 2\left(\frac{mV_S^2}{k_BT} - 1\right)^{-1}$

(b) Given data: $n_d = 2\left(\frac{4.82\times10^{-26}\times 346^2}{1.38\times10^{-23}\times 293} - 1\right)^{-1}$

$$= 4.87 \simeq 5$$

(c) Probably not. Air is mainly N_2 & O_2 i.e. diatomic. So the discussion in Sec 1.3 (Lec 1) would have suggested $n_d = 7$. The reason will be revealed in Lec 6.
