

Structure of Matter: Introduction to the Course

This course is about explaining the properties of matter by examining the underlying mechanism.

Consider some phenomenon on the everyday, or **macroscopic**, scale. Now magnify it by $\sim 10^9$ and look at it again on the atomic, or **microscopic**, scale.

It will look totally different.

How do micro-world properties translate into macro-world properties?

Explaining the properties of the macro world in terms of the micro behaviour is

EXTREMELY HARD

because

- things often depend on complicated details rather than simple unifying principles
- many unfamiliar concepts are involved
- the maths can be pretty scary
- the micro world is inherently weird

You cannot learn the whole subject at one attempt.

Instead you need to go over it lots of times.

The *Structure of Matter* course is the first attempt.

In later years you will study aspects of the subject in more detail, e.g., in the 2nd year there are courses on:

Thermodynamics

Statistical Physics

Applications of Quantum Mechanics

Electrons in Solids

In this course we will:

- use simplified models
- encounter some of the most widely used theoretical concepts
- gain a qualitative understanding of some fundamental ideas
- obtain fairly good estimates of certain parameters

“If, in some cataclysm, all of scientific knowledge were to be destroyed, and only one sentence passed on to the next generation of creatures, what statement would contain the most information in the fewest words? I believe it is...

All things are made of atoms – little particles that move around in perpetual motion, attracting each other when they are a little distance apart, but repelling upon being squeezed into one another.

In that one sentence there is an enormous amount of information”

Feynman Lectures, Volume 1, page 1-2

Atoms are little

Estimate size of atom, e.g., Iron:

$$\rho = \text{density} = 7870 \text{ kg m}^{-3}$$

$$\text{Atomic weight} = 55.8$$

$$m_{\text{atom}} = \text{mass of one Iron atom}$$

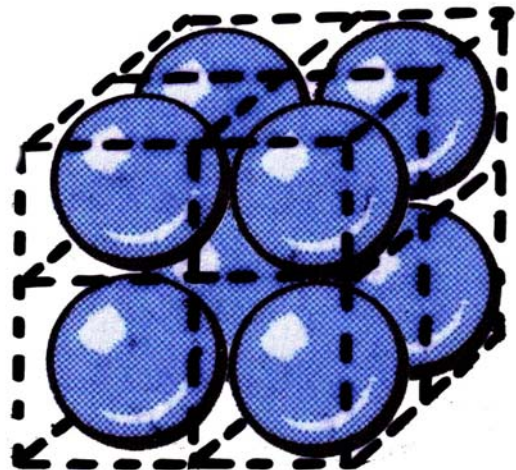
$$= 55.8 \times 1.66 \times 10^{-27} = 9.33 \times 10^{-26} \text{ kg}$$

$$n = \text{number density of atoms}$$

$$= \text{number of atoms per unit volume}$$

$$= \rho / m_{\text{atom}} = 8.44 \times 10^{28} \text{ atoms m}^{-3}$$

Assume spherical atoms (diameter d) arranged in simple lattice.



$$V = \text{vol occupied by } N \text{ atoms} = N \times d^3$$

$$\rightarrow d = (V/N)^{1/3} = n^{-1/3} = 2.3 \times 10^{-10} \text{ m}$$

Digression about moles

A mole of stuff is 6.02×10^{23} particles (atoms or molecules) of that stuff.

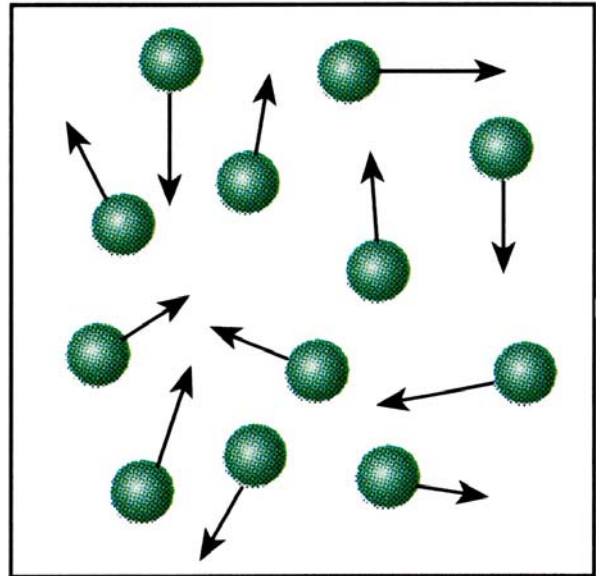
6.02×10^{23} is called Avogadro's number (N_A).

If the particles are atoms of atomic weight M then 1 mole of them has a mass of:

$$\begin{aligned} & M \times 1.66 \times 10^{-27} \times 6.02 \times 10^{23} \\ &= M \times 10^{-3} \text{ kg} \\ &= M \text{ g} \end{aligned}$$

Atoms are in perpetual motion

e.g., in a gas



Direction of motion and speed vary randomly from one atom* to another.

This random, **thermal**, motion occurs in all matter.

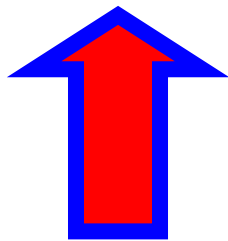
Average k.e. of atoms \propto temperature

* In many gases, e.g. N_2 , the relevant particles are molecules, not atoms.

Temperature is a macro-world concept.

At the micro-level it translates into energy.

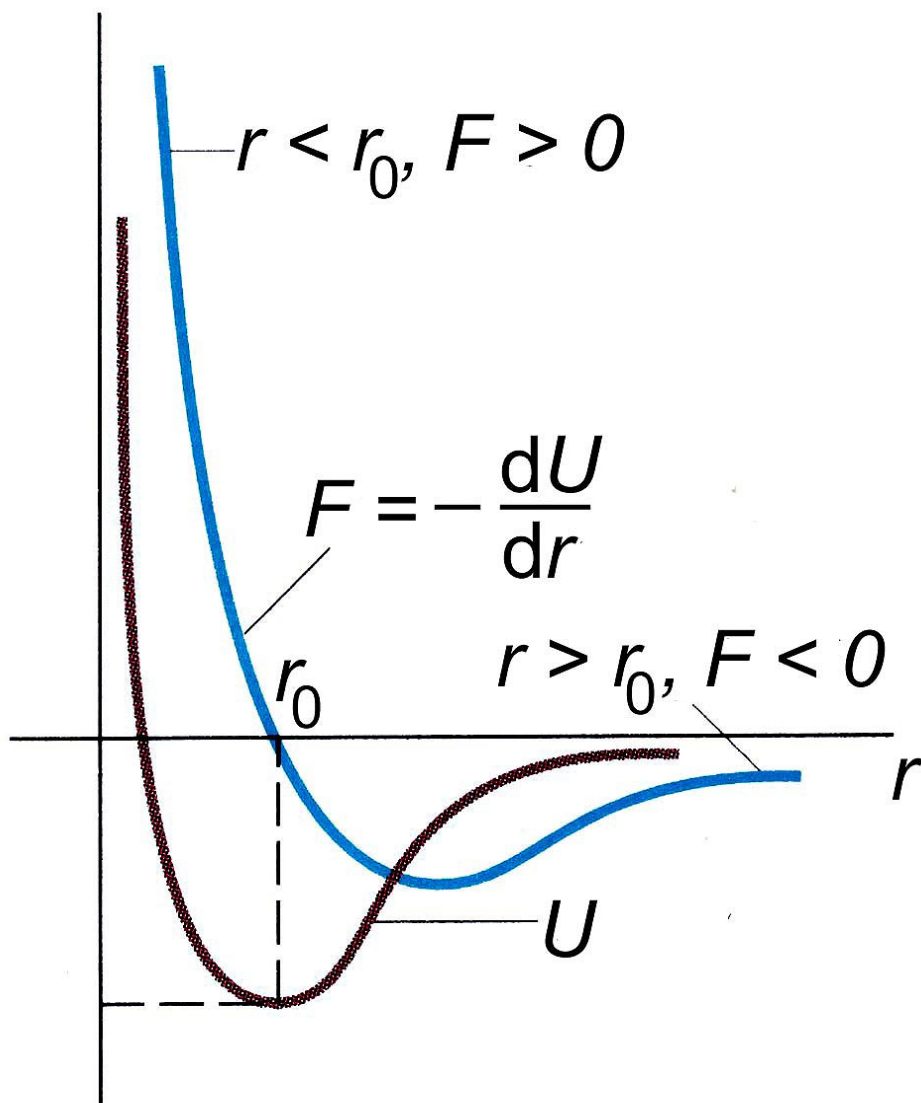
Energy = constant \times temperature



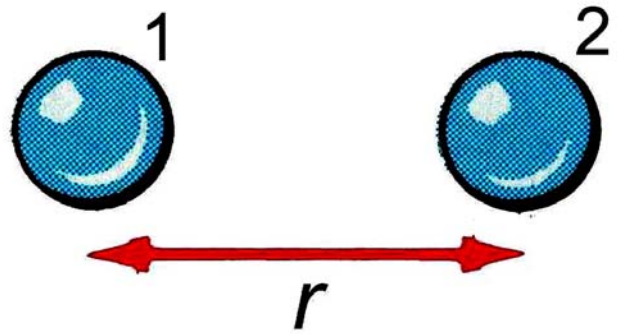
This conversion factor is a key parameter in going from the micro-world to the macro-world. But what is it?

There is an attractive force between atoms some distance apart, but a repulsive force when they are very close

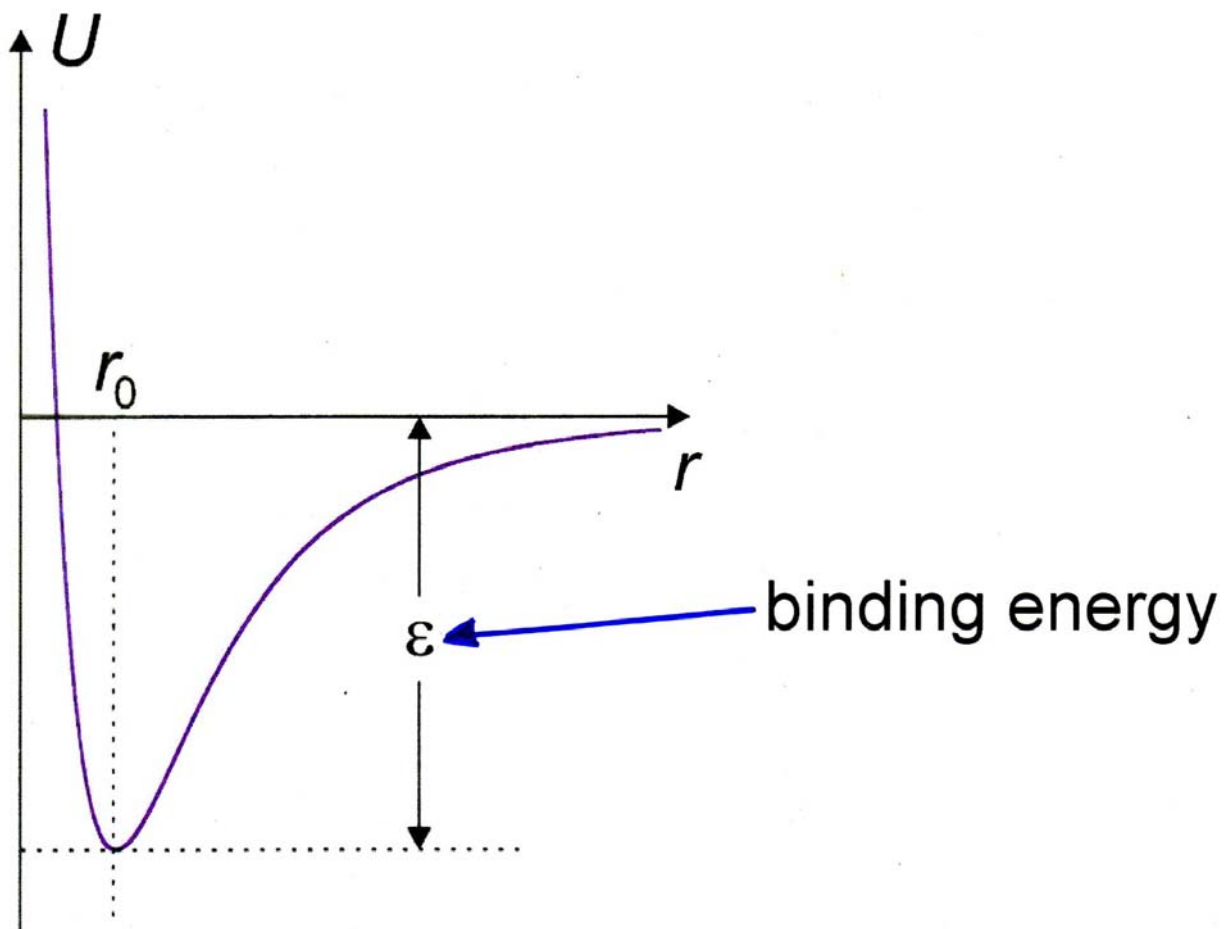
Convenient to use potential energy (U)



Two atoms
distance r apart.
Atom 2
experiences force
due to atom 1.

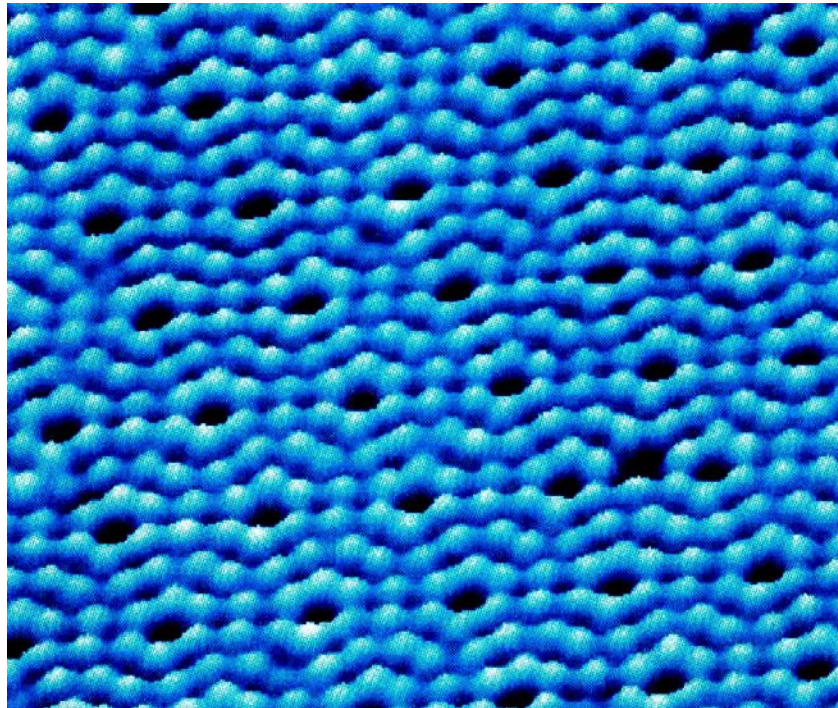


If atom 2 had zero k.e. it would be fixed
at eqm position ($r = r_0 =$ bottom of
potential well)



Lots of atoms with zero k.e. will be tightly bound in a regular lattice

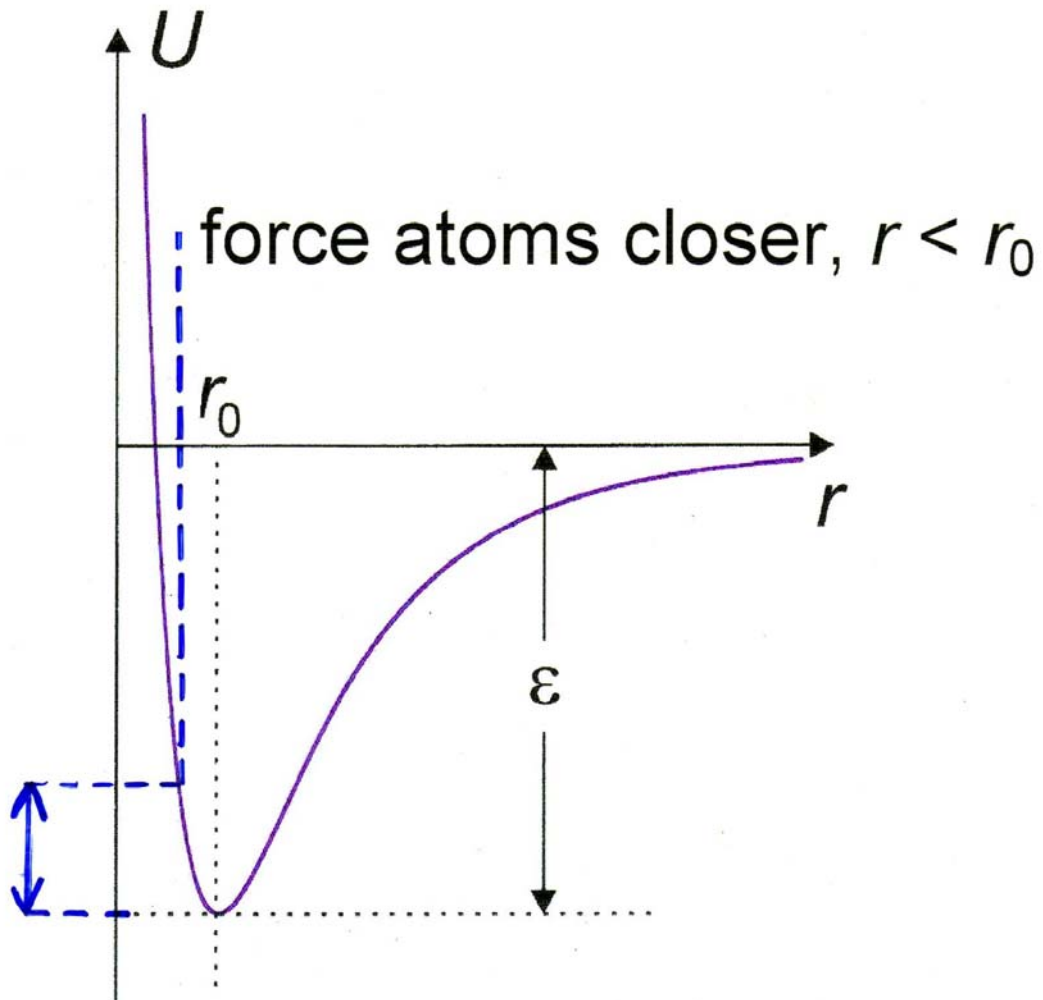
→ **solid**



Previous estimate of atomic diameter was actually for r_0 i.e., $r_0 \sim \text{few} \times 10^{-10}$ m.

In fact, atoms are packed more closely than in our simple arrangement. In close packed arrangement each atom touches (i.e., is bound to) 12 others.

Very difficult to reduce separation of the atoms much below equilibrium value, r

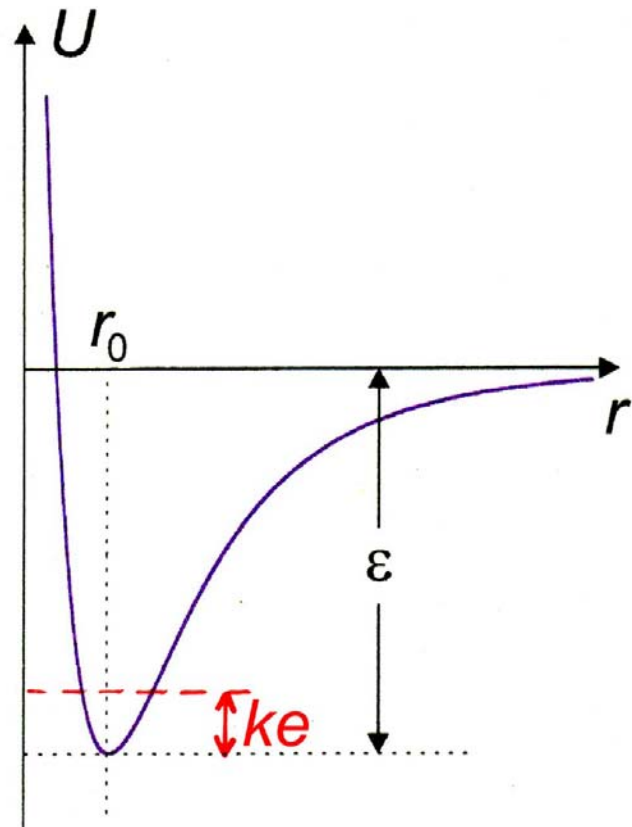


atomic p.e. increases dramatically
→ need large force to compress solid

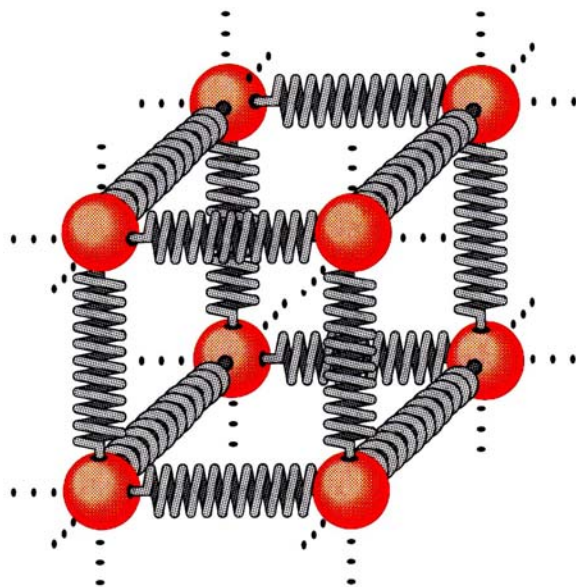
This is why solids are hard.

Actually, atoms are in ceaseless thermal motion, i.e., atoms have finite k.e.

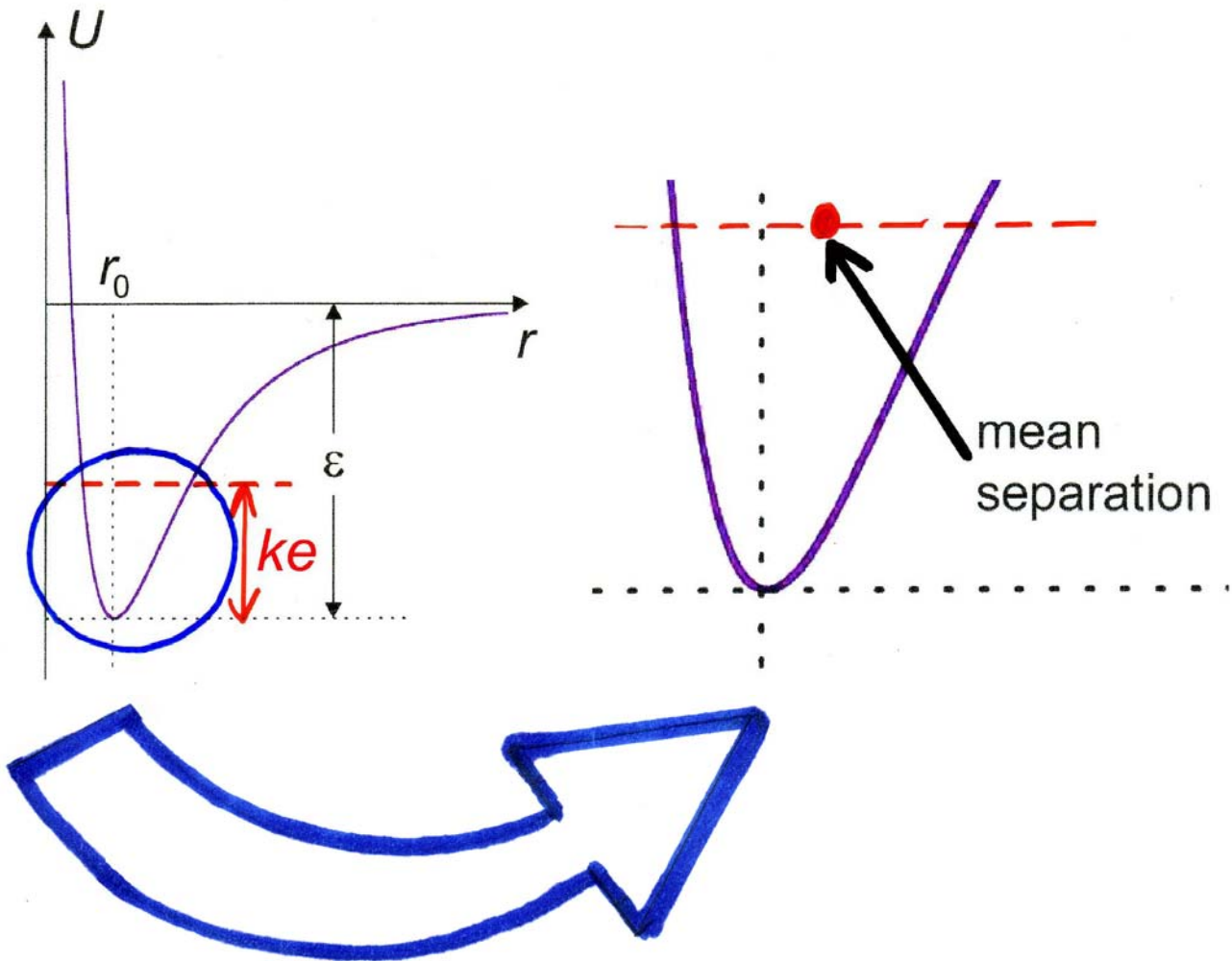
If $k.e. \ll \epsilon$
atoms oscillate
about bottom
of potential
well



Atoms in solid
vibrate about
equilibrium
positions



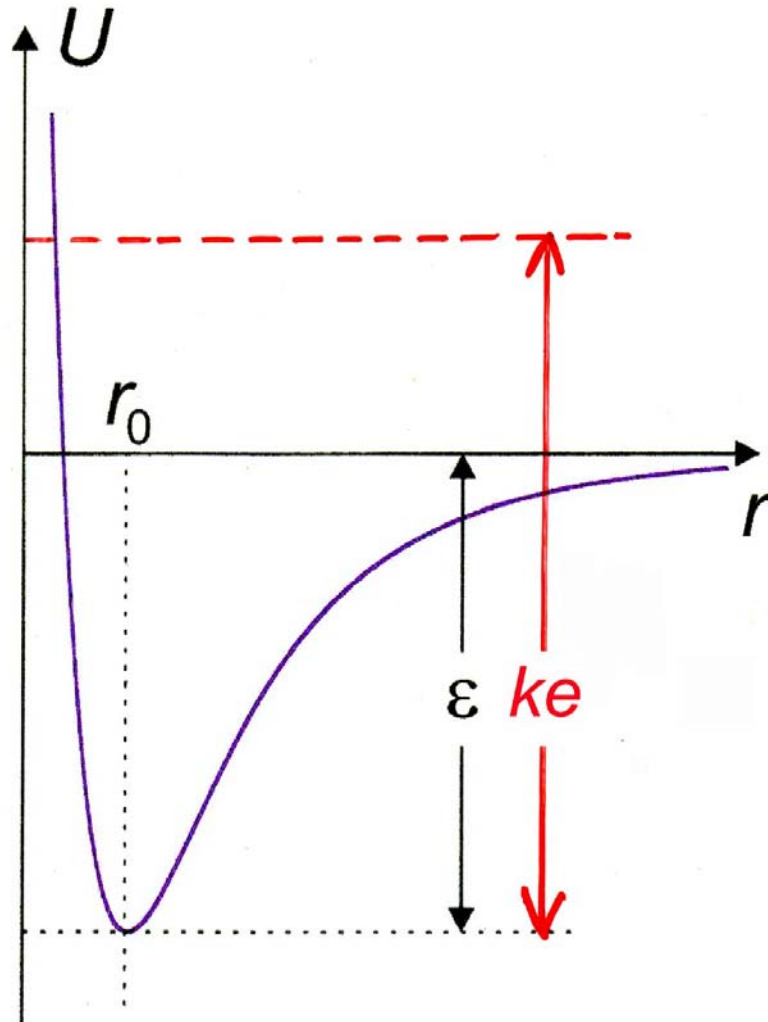
Increase T (i.e., increase k.e. of atoms)
 → amplitude of oscillations increases.



Mean separation of atoms $> r_0$ because potential well is not symmetric.

Solids expand when T is raised.

Increase T a lot \rightarrow k.e. of atoms $> \varepsilon$

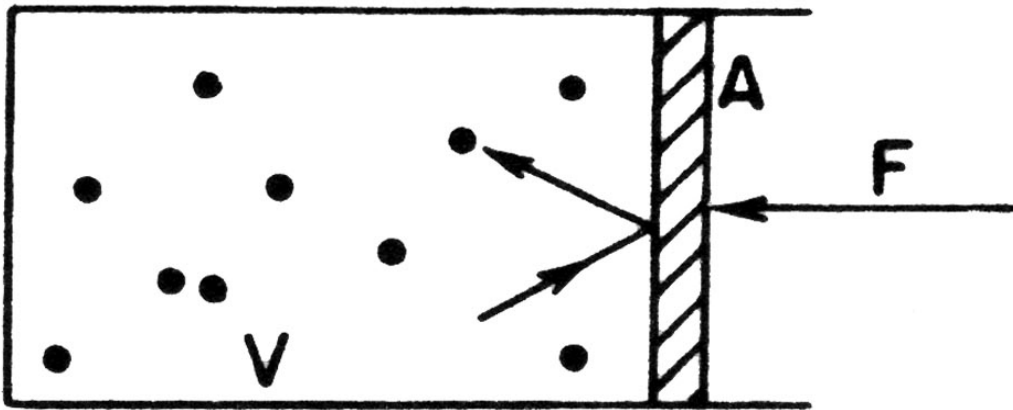


Atoms no longer bound
 \rightarrow free to move about
 \rightarrow **gas**

Thermal motion not oscillatory but translational.

Gas atoms constantly colliding with wall of container \rightarrow force on wall.

P = pressure
= force per unit area due to atomic bombardment



Increase N (= number of atoms),
keep V (=volume) and T constant.

→ more collisions with wall per second

→ **P increases**

Increase T , keep N and V constant.

→ k.e. of atoms increases

→ atoms hit wall harder

→ **P increases**

Increase V , keep N and T constant.

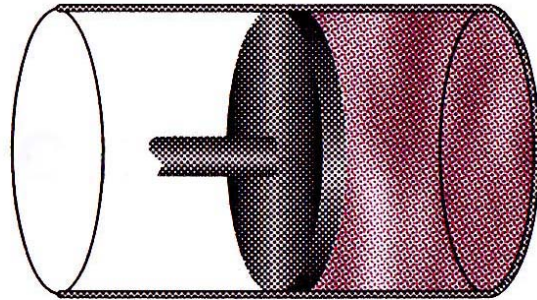
→ atoms spread out more

→ fewer collisions with wall per second

→ **P decreases**

Increase T , keep P and N constant

e.g. gas in
cylinder with
piston



No net force on piston in equilibrium:

$$P_{\text{inside}} = P_{\text{outside}} = \text{atmospheric pressure}$$

Increase T inside

→ P inside increases

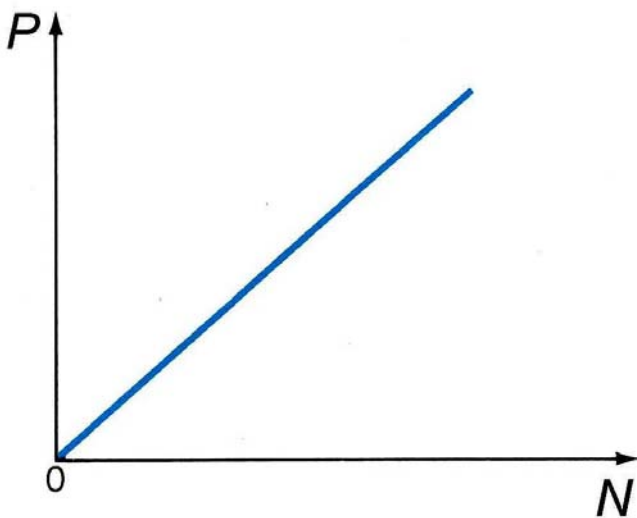
→ **V increases** until $P =$ atmospheric pressure again

Both gases and solids expand when T is raised, but for different reasons.

This is an example of a macroscopic property depending on micro-level details rather than simple unifying principles.

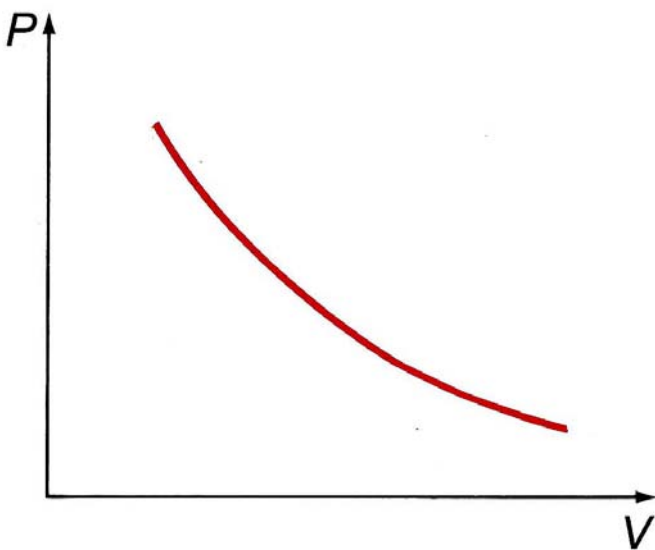
Experimental result for dilute (i.e., low density) gases:

Vary N , keep V and T constant



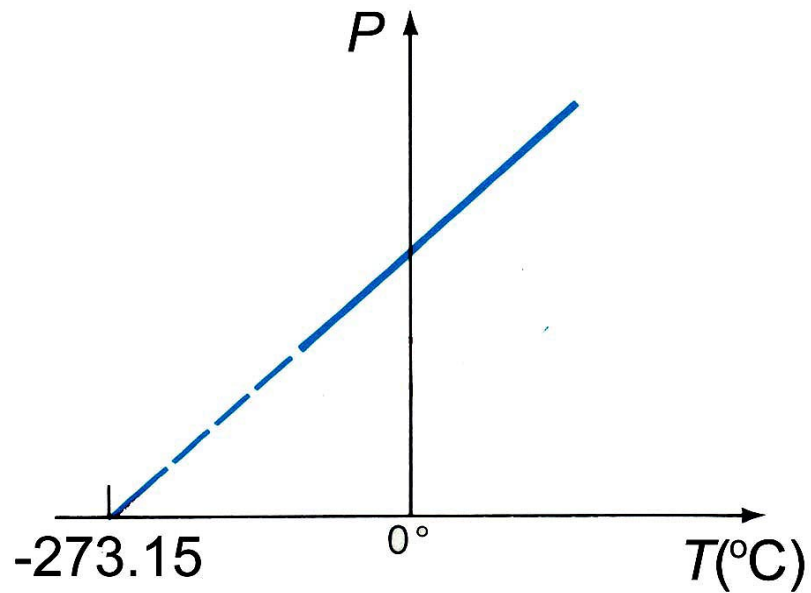
$$P \propto N$$

Vary V , keep N and T constant

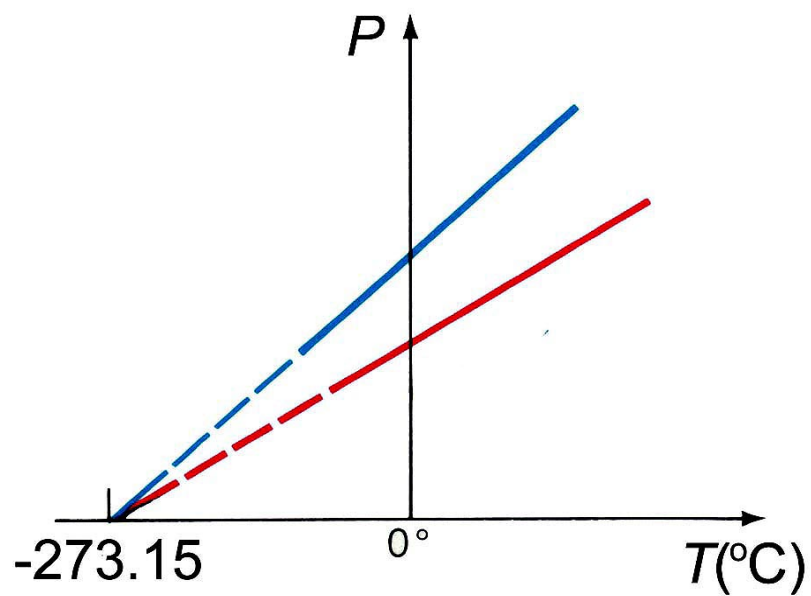


$$P \propto 1/V$$

Vary T , keep N and V constant

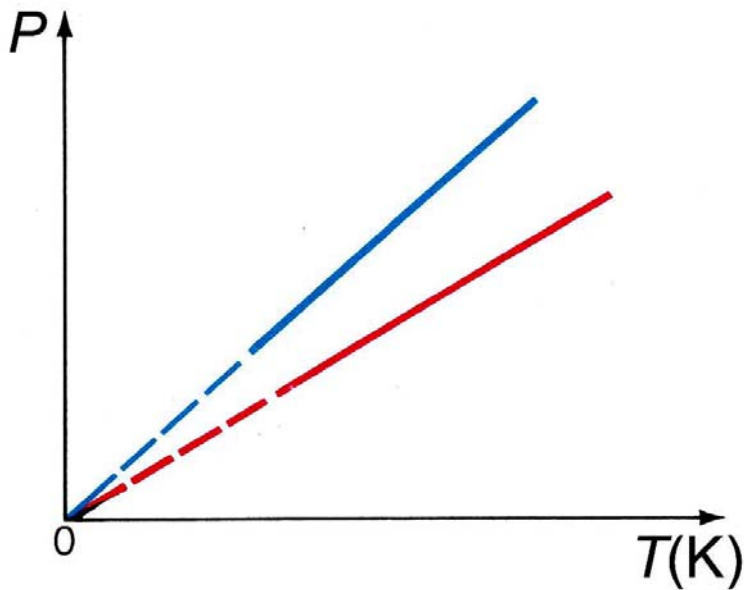


Use different gas



Absolute temperature (in K):

$$T(\text{K}) = T(^{\circ}\text{C}) + 273.15$$



$$P \propto T$$

Atoms would have zero k.e. at absolute zero, 0 K.

These experimental observations can be summarized in:

The Ideal Gas Equation of State
$$PV = Nk_B T$$

k_B = constant of proportionality
= Boltzmann's constant
 $= 1.38 \times 10^{-23} \text{ J K}^{-1}$

IGEOS can be written in terms of moles

$$PV = \frac{N}{N_A} N_A k_B T = n_{\text{moles}} RT$$

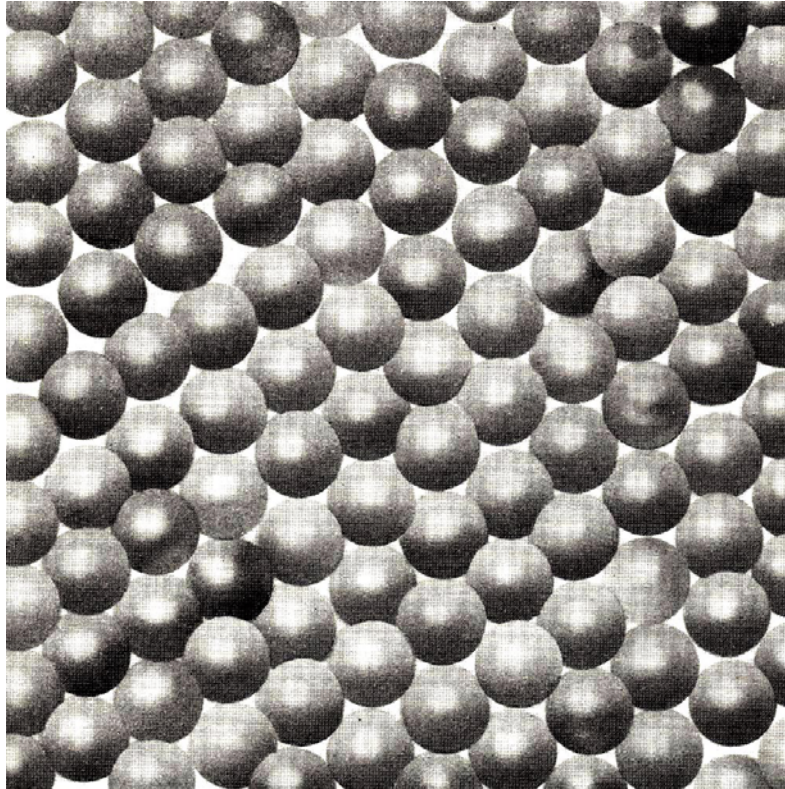
R = The Universal Gas Constant
 $= 8.31 \text{ J K}^{-1} \text{ mol}^{-1}$

Our simple model based on the p.e. of interacting atoms gives good qualitative insights.

But it has many shortcomings:

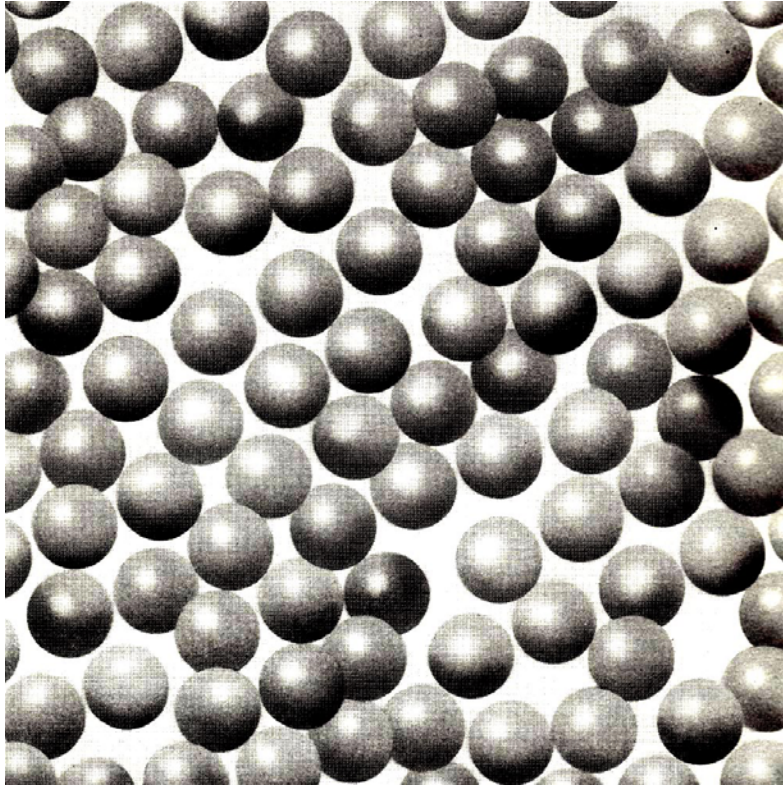
- quantum effects are important
- particles other than atoms can be involved, e.g., molecules
- particles can have internal structure, e.g., diatomic molecules consist of two atoms joined together
- particles can have a range of energies, i.e., not all at the same height above bottom of the potential well
- **liquids** exist, as well as solids and gases

Solid



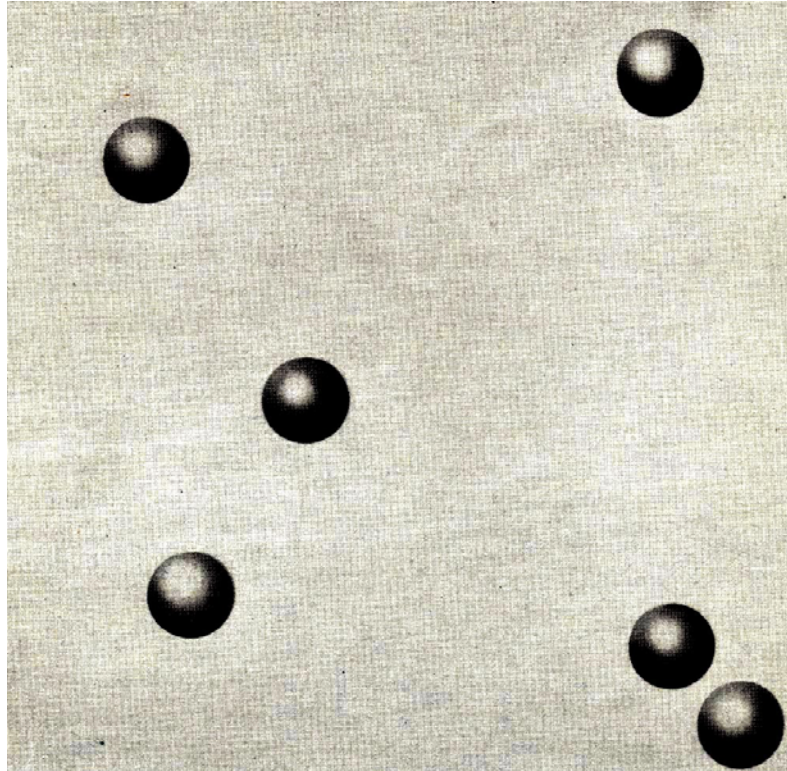
- Each particle bound to ~ 12 others, in regular lattice
- Particle number density very high
- Particles oscillate about equilibrium position

Liquid



- Each particle bound to several others, but compared to solids some bonds broken
- Particle arrangement is irregular
- Particle number density only slightly lower than solid
- Particles can move about
- Particle motion partly oscillatory, partly translational

Gas



- Particles not bound
- Particle number density very much lower than solid
- Particles have translational motion

In solid each atom joined to ~12 others
In liquid each atom joined to ~10 others
In gas each atom joined to 0-1 others

Each bond joins two atoms

A collection of N atoms involves:

$N \times 12 \times \frac{1}{2}$ bonds if it is a solid

$N \times 10 \times \frac{1}{2}$ bonds if it is a liquid

0 or $N \times \frac{1}{2}$ bonds if it is a gas

Vaporizing (liquid \rightarrow gas) involves breaking $\sim 5N$ bonds.

Melting (solid \rightarrow liquid) involves breaking $\sim N$ bonds.

Therefore:

latent heat of vaporization (= amount of energy needed to vaporize 1 kg of given substance)

must be much greater than

latent heat of fusion (= amount of energy needed to melt 1 kg of substance).

e.g. Oxygen

$$\text{lat. ht. vap.} = 2.13 \times 10^5 \text{ J kg}^{-1}$$

$$\text{lat. ht. fus.} = 1.38 \times 10^4 \text{ J kg}^{-1}$$

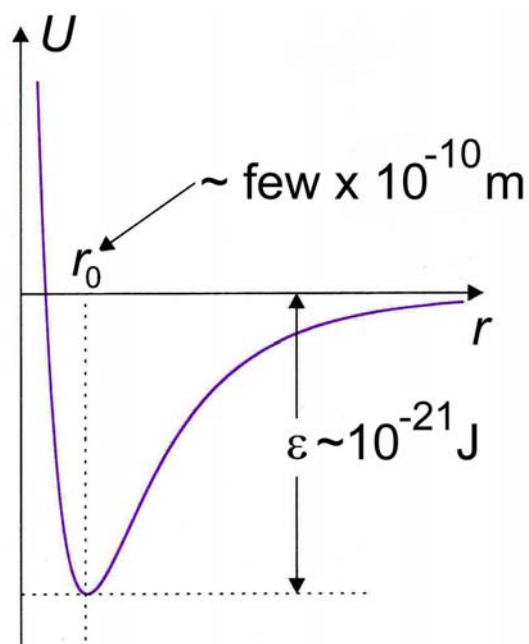
Number of atoms in 1 kg of Oxygen
(atomic weight 16.0)
 $= 1/(16.0 \times 1.66 \times 10^{-27}) = 3.77 \times 10^{25}$

Therefore vaporizing 1 kg of Oxygen
involves breaking $\sim 1.9 \times 10^{26}$ bonds.

Requires 2.13×10^5 J (lat. ht. vap.)

ε = binding energy
 = energy associated with each bond
 $\approx 2.13 \times 10^5 / 1.9 \times 10^{26}$
 $\sim 1.1 \times 10^{-21}$ J (~ 0.007 eV)

So we have
estimates for
scales on p.e.
curve



At absolute zero all atoms would have
k.e. = 0.

At boiling point the atoms cease to be
bound, i.e., k.e. $\approx \varepsilon$

Can estimate temperature/energy
conversion factor:

$$\varepsilon = \text{constant} \times T_{\text{boiling}}(\text{K})$$

For Oxygen:

$$\varepsilon \approx 1.1 \times 10^{-21} \text{ J}$$

$$T_{\text{boiling}} = 90.2 \text{ K}$$

$$\rightarrow \text{constant} \approx 1.3 \times 10^{-23} \text{ J K}^{-1}$$

**The temperature/
energy conversion
factor is Boltzmann's
constant.**