SoM: Classwork IV, Answers

1. F_0 = force acting to left on C-D.
   = $2 \times \gamma l$ (γl from each surf of the soap film)

Work done = $(F_0 + dF/dx = F_0 dx$ (ignoring 2nd order term $dF\,dx$)

= $2\gamma l\, dx$

dA = increase in surf area
= $2 \times l\, dx$ (each surf increases by $l\, dx$)

→ increase in energy = work done = $\gamma\, dA$

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2. Atoms in the bulk of the liquid are bound to ~10 others (see Intro Lec.). Atoms at the surf are only bound to ~5. Increasing the area of the surf- involves transferring atoms from the bulk of the liquid to the surface. The energy is used to break the atomic bonds.

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3. Surface area at radius r : $4\pi r^2$

\- - - - - - - $r+dr$: $4\pi(r+dr)^2$
$= 4\pi(r^2 + 2rdr + dr^2)$

$\therefore dA = 4\pi \times 2r\,dr$ (ignoring 2nd order term dr^2)

$\therefore$ work done = $\gamma\, dA = 8\pi\gamma r\, dr$

But work done = $F\,dr \rightarrow F = 8\pi\gamma r$

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4. Total inward force on surface

$$\underbrace{P_{out} \times 4\pi r^2}_{P_{outside}} - \underbrace{P_{in} \times 4\pi r^2}_{P_{inside}} + \underbrace{8\pi\gamma r}_{\text{surf ten.}}$$

But total force $= 0 \rightarrow P_{in} = P_{out} + \frac{2\gamma}{r}$

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5. $P = P_0 + \rho g d$ (Eq 9.1.1, Lec 9).

At depth $d = 1m \rightarrow P = \underbrace{1.01 \times 10^5}_{P_0} + \underbrace{10^3 \times 9.81 \times 1}_{\rho g d} = 1.11 \times 10^5 \, Nm^{-2}$

Pressure required $= P_{in} = 1.11 \times 10^5 + \frac{2 \times 0.072}{10^{-4}}$

$= 1.12 \times 10^5 \, Nm^{-2}$
