Structure of Matter
First Year Physics

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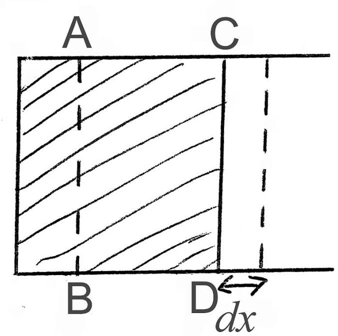
19.05.05

Classwork IV
Surface Tension

Information needed for this Classwork

1 atmosphere = 1.01×10^5 N m^{-2}.
Acceleration due to gravity: $g = 9.81$ m s^{-2}
Density of water: $\rho = 10^3$ kg m^{-3}
Surface tension of water: $\gamma = 0.0728$ N m^{-1}

Liquids try to minimize their surface area; a liquid surface is like a stretched elastic membrane. The force responsible for this is *surface tension.*



Consider a soap film on a wire frame, as shown. The surface to the left of the imaginary line AB pulls the line to the left, but the surface to right exerts an equal force to the right. The force acts perpendicular to the line and is proportional to the length, l, of the line; it can be written $F = \gamma l$, where γ is the surface tension (SI unit N m^{-1}).

1. Assume section CD of the wire frame is movable. To keep it in place a force F_0 must be applied equal to the surface force pulling to the left. Increasing the applied force an infinitesimal amount dF will result in section CD moving an infinitesimal distance dx to the right. Show that the work done is $2\gamma l \mathrm{d}x$. (This is analagous to the quasi-static work done by a piston compressing a gas.) Hence, show that the increase in energy of the soap film can be written $\gamma \mathrm{d}A$, where dA is the change in the surface area. (Hint: remember a soap film has two surfaces.)

2. It requires energy $\gamma \mathrm{d}A$ to increase the area of a fluid surface. You might think that at the microscopic (i.e., atomic) level this energy is used to pull the atoms apart and increase their separation. But, in fact, the average interatomic separation doesn't change; liquids are incompressible, so the volume of the soap film is constant. The thickness of the film simply decreases when the area increases.

   So, at the microscopic level, where does the energy go?

3. The surface of a spherical bubble (of radius r) in a liquid (e.g., champagne) experiences a net inward force F_c towards the centre. This force can be found by calculating the work needed to increase the size of the bubble. If an infinitesimally larger outward force is applied to the surface the radius will increase by $\mathrm{d}r$ and the work done will be $F_c \mathrm{d}r$. Find an expression for $\mathrm{d}A$, the increase in surface area corresponding to an increase in radius of $\mathrm{d}r$, and hence show that $F_c = 8\pi\gamma r$. (Hint: remember that this type of bubble, more properly called a *cavity*, has only one surface.)

4. In addition to this surface tension force, the surface is also acted on by P_{out}, the pressure of the surrounding liquid, and P_{in}, the pressure of the gas inside the bubble. Show that $P_{in} = P_{out} + \dfrac{2\gamma}{r}$. (This equation is called the Laplace equation.)

5. It is proposed to aerate a large open tank of water by pumping air into the water at a depth of 1 m below the surface. What air pressure is required to produce bubbles of radius 0.1 mm. Assume that the pressure at the surface of the water (i.e., at the top of the tank) is 1 atmosphere.

Numerical Answer

5) 1.12×10^5 N m^{-2}