SoM : Classwork III, Answers

1. Probability of any one mol. being between v_x and $v_x + dv_x$ is $f(v_x)dv_x$.

Total no of such mol's in vol $V = N \times \text{prob} = N f(v_x)\, dv_x$

$\therefore$ no density of such mol's $= \left(\frac{N}{V}\right) f(v_x)\, dv_x$ ← n

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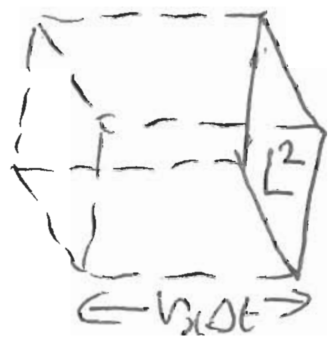
2. Only mol's within distance $v_x \Delta t$ reach wall in Δt.



Vol containing these mol's $= L^2\, v_x \Delta t$

$$N_{coll} = \underbrace{n f(v_x)\, dv_x}_{\text{no density}} \times \underbrace{L^2 v_x \Delta t}_{\text{vol.}}$$

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3. The factor of ½ arose because we used $n = N/V$ to find the total no of mol's in the vol $L^2 v_x \Delta t$, 50% of which are moving away from wall. Here, however, we consider $n f(v_x) dv_x$ = no. density of mol's between v_x & $v_x + dv_x$, with $v_x > 0$ i.e. we have already excluded those moving away from the wall.

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4. Momentum imparted to wall in Δt by mol's between v_x & $v_x + dv_x$ is

$$\Delta p = N_{coll} \times 2 m v_x = 2 L^2 n m v_x^2 f(v_x)\, dv_x\, \Delta t$$

dP = contribution to P from these mol's

$$= \frac{1}{L^2} \frac{\Delta p}{\Delta t} = 2nmv_x^2 f(v_x) dv_x$$

So $P = \int_{v_x=0}^{v_x=\infty} 2nmv_x^2 f(v_x) dv_x$

$$= 2nmA \int_0^\infty v_x^2 e^{-\alpha v_x^2} dv_x$$

$$= 2nmA \frac{1}{4} \left(\frac{\pi}{\alpha^3}\right)^{1/2}$$

$$= \frac{\pi^{1/2}}{2} nm \left(\frac{m}{2\pi k_B T}\right)^{\frac{1}{2}} \left(\frac{2k_B T}{m}\right)^{3/2}$$

$$= \frac{1}{2} nm \frac{2k_B T}{m} = nk_B T$$
