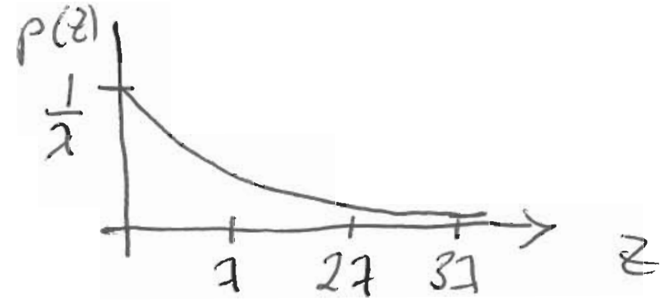
SoM: Classwork II, Answers

1. Eq 4.2.2 → $p(z) = \frac{mg}{k_B T} e^{-mgz/k_B T}$

   i.e. $\lambda = \frac{k_B T}{mg}$

2.



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3 (a) prob $= \int_0^\lambda p(z)\,dz = \frac{1}{\lambda}\int_0^\lambda e^{-z/\lambda}\,dz$

$= \frac{1}{\lambda}\left[-\lambda e^{-z/\lambda}\right]_0^\lambda = -\left[e^{-1} - e^{0}\right]$

$= 1 - e^{-1} = 0.63$

(b) prob $= \int_\lambda^{2\lambda} p(z)\,dz = \frac{1}{\lambda}\left[-\lambda e^{-z/\lambda}\right]_\lambda^{2\lambda}$

$= e^{-1} - e^{-2} = 0.23$

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4 $\int_0^\infty p(z)\,dz = \frac{1}{\lambda}\left[-\lambda e^{-z/\lambda}\right]_0^\infty = -\left[e^{-\infty} - e^{0}\right] = 1$

The molecule must be at some height i.e. prob of being between $z=0$ & $z=\infty$ is 1 (= certainty).

5. $\lambda = \dfrac{1.38\times10^{-23}\times 293}{4.82\times10^{-26}\times 9.81} = 8.55\times10^{3}\,m$

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6. Prob that mol is below z^* is

$$= \int_0^{z^*} p(z)\,dz = 1 - e^{-z^*/\lambda}$$

i.e. $1 - e^{-z^*/\lambda} = 0.999 \rightarrow -z^*/\lambda = \ln(0.001)$

$\therefore z = 6.91\lambda = 59.1\,km$

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7 99.9% of mol's are within 59 km of the surface of the Earth. 59km $\ll$ radius of Earth $\therefore$ assuming g = const is OK.
