Structure of Matter
First Year Physics

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10.05.05

Classwork II
The Isothermal Atmosphere

Information needed for this Classwork

Boltzmann's constant: $k_B = 1.38 \times 10^{-23}$ J K^{-1}.
Absolute zero = -273^OC.
Acceleration due to gravity: $g = 9.81$ m s^{-2}.

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1. The probability that a given molecule in an isothermal atmosphere is between heights z and $z + \mathrm{d}z$ can be written $p(z)\mathrm{d}z$, where $p(z) = \frac{1}{\lambda}\mathrm{e}^{-z/\lambda}$. Write down an expression for λ in terms of T, the temperature, and m, the mass of a molecule.
2. Sketch $p(z)$ for $0 < z < 3\lambda$.
3. The probability that a molecule is located between $z = z_1$ and $z = z_2$ is $\int_{z_1}^{z_2} p(z)\ \mathrm{d}z$.
   (a) Calculate the probability that a given molecule is between $z = 0$ and $z = \lambda$.
   (b) Calculate the probability that a given molecule is between $z = \lambda$ and $z = 2\lambda$.
4. Show that $\int_0^\infty p(z)\ \mathrm{d}z = 1$, and interpret this result.
5. Calculate λ for $T = 20^o$ C and $m = 4.82 \times 10^{-26}$ kg (the average mass per molecule in air).
6. There is a 99.9% probability that any given molecule is located below a certain height z^*. Calculate the value of z^* for the parameters given in Q 5.
7. In deriving the isothermal atmosphere model we assumed that g, the acceleration due to gravity, was independent of height. Was this a reasonable assumption?

Numerical Answers
3a) 0.63; 3b) 0.23; 5) 8.55 km; 6) 59.1 km