SoM: Classwork I, Answers

1. $\rho = mn$, no density $= N/V$

$$= \frac{mP}{k_B T} = \frac{4.82\times10^{-26}\times1.01\times10^{5}}{1.38\times10^{-23}\times293} = 1.20\ \text{kgm}^{-3}$$

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2. P_0 = pressure outside $= n_0 k_B T_0$ ← ρ_0/m

P_i = pressure inside $= n_i k_B T_i$ ← ρ_i/m

But $P_i = P_0 \Rightarrow \rho_i = \rho_0 T_0 / T_i$

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3. mass of air inside $= \rho_i V = \rho_0 V \frac{T_0}{T_i}$

Total mass of balloon (air + load) $= \rho_0 V \frac{T_0}{T_i} + m_L$

mass of air displaced $= \rho_0 V$

Takes off if $\rho_0 V \geq \rho_0 V \frac{T_0}{T_i} + m_L$

$$\rightarrow 1 - \frac{m_L}{\rho_0 V} \geq \frac{T_0}{T_i} \rightarrow T_i \geq \frac{T_0}{1 - m_L/\rho_0 V}$$

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4. $T_{min} = \frac{T_0}{1 - m_L/\rho_0 V} = \frac{293}{1 - 300/(1.20\times10^{3})}$

$= 391\ \text{K}\ (= 118^\circ\text{C})$

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5. Ideal gas: $U = \frac{n_d}{2} N k_B T$ n_d = no of degrees of freedom

$$= \frac{n_d}{2} PV$$

balloon $P = \text{const}$ $(= P_{ext})$ & $V = \text{const}$

$\therefore U = \text{const} \rightarrow U_i / U_0 = 1$

(As T rises the average energy of the molecules also rises, but some escape from the vol & so N goes down $\rightarrow U = \text{const}$)

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6. You can't calculate U_0 & U_1 without knowing n_d, & you're not given this information.
