Structure of Matter
First Year Physics

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Classwork I
Hot Air Balloons

Information needed for this Classwork

Boltzmann's constant: $k_B = 1.38 \times 10^{-23}$ J K^{-1}.
Absolute zero $= -273^O$C.
1 atmosphere $= 1.01 \times 10^5$ N m^{-2}.

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1. Air consists of a mixture of gases, notably Nitrogen and Oxygen. The average mass per molecule in air is 4.82×10^{-26} kg. Assuming that air can be treated as an ideal gas, calculate the density of air (in kg m^{-3}) at a pressure of 1 atmosphere and a temperature of 20^o C.

2. A container of fixed volume is full of air. There is a small hole in the container, as a result of which the air pressure inside is always equal to the external air pressure. However, the temperature of the air inside can be raised above the external air temperature. Show that the density of the air inside the container is $\rho_1 = \rho_0 T_0/T_1$, where ρ_0 and T_0 are the external air density and temperature respectively, and T_1 is the internal air temperature.

3. Consider a hot air balloon to be a fixed volume container of the type discussed in Q. 2. Archimedes' Principle tells us that the balloon will rise if the mass of air displaced by the balloon exceeds the total mass of the balloon (i.e., the mass of air inside, plus the mass of the load and the material of the balloon itself). Assuming that the volume of the air inside is V, and that the volume of the load is negligible, show that the balloon will take off with a load of mass m_L (which includes the mass of the material of the balloon) if the temperature of the air inside satisfies the condition:

$$T_1 \geq T_0 \left(1 - \frac{m_L}{\rho_0 V}\right)^{-1}$$

4. Calculate the minimum temperature T_1 for a hot air balloon of volume $V = 1000$ m^3 with a load of mass $m_L = 300$ kg to take off, assuming that the external air is at a pressure of 1 atmosphere and a temperature of 20^oC.

5. Before the air in the ballon was heated (i,e., while still at the ambient temperature of 20^oC) the internal energy of the air in the balloon was U_0. After the temperature was raised to the value found in Q. 4 the internal energy of the air in the balloon was U_1. Without calculating the separate values U_0 and U_1, find the ratio U_1/U_0.

6. Why are you asked not to calculate the separate values U_0 and U_1 in Q. 5?

Numerical Answers
1) 1.20 kg m^{-3}; 4) 391 K (=118 ^{o} C); 5) 1