Structure of Matter

Problem Sheet 1 **Answers**

1. (a) $N = \dfrac{PV}{k_B T} = \dfrac{0.5 \times 10^3}{1.38 \times 10^{-23} \cdot 300} = 1.22 \times 10^{25}$

   (b) $V = \dfrac{N_m RT}{P} = \dfrac{3 \times 8.31 \times 263}{10^2} = 65.6 \text{ m}^3$

   (c) $V = \dfrac{8.31 \times 273}{1.01 \times 10^5} = 0.0224 \text{ m}^3 = 22.4 \text{ litres} = 22{,}400 \text{ cc}$

2. $P = Nk_BT/V = nk_BT$, so $P_e = P_i = 5 \cdot 10^{19} \times 1.38 \cdot 10^{-23} \times 10^8 = 6.9 \cdot 10^4$ Pa

   $P_{Total} = P_e + P_i = 1.38 \cdot 10^5$ Pa $= 1.37$ atmospheres

   $U_i = U_e = \frac{3}{2} Nk_BT = \frac{3}{2} PV = \frac{3}{2} \times 6.9 \cdot 10^4 \times 10^3 = 1.03 \cdot 10^8$ J

   $U_{Total} = U_i + U_e = 2.07 \cdot 10^8$ J

3. (a)

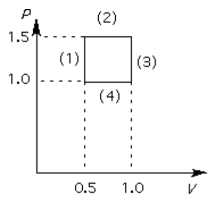

Figure 1: default

   (b) Using $\dfrac{P_1V_1}{T_1} = Nk_B = \text{constant}$, so

   (1) $\dfrac{1 \cdot 0.5}{300} = \dfrac{1.5 \cdot 0.5}{T_1} \quad \rightarrow \quad T_1 = 450 \text{ K} \quad \rightarrow \Delta T = 150 \text{ K}$

   (2) $\dfrac{1.5 \cdot 0.5}{450} = \dfrac{1.5 \cdot 1}{T_2} \quad \rightarrow \quad T_2 = 900 \text{ K} \quad \rightarrow \Delta T = 450 \text{ K}$

   (3) $\dfrac{1.5 \cdot 1}{900} = \dfrac{1 \cdot 1}{T_3} \quad \rightarrow \quad T_3 = 600 \text{ K} \quad \rightarrow \Delta T = -300 \text{ K}$

   (4) $\dfrac{1 \cdot 1}{300} = \dfrac{1 \cdot 0.5}{T_4} \quad \rightarrow \quad T_4 = 300 \text{ K} \quad \rightarrow \Delta T = -300 \text{ K}$

   (c) $\Delta U = \frac{3}{2} Nk_B(T_1 - T_0) = \frac{3}{2}(P_1V_1 - P_0V_0)$ so,

   (1) $\Delta U = \frac{3}{2}(1.5 * 1.01 \times 10^5 * 0.5 - 1.0 * 1.01 \times 10^5 * 0.5) = 3.79 \times 10^4$ J

   (2) $\Delta U = \frac{3}{2}(1.5 * 1.01 \times 10^5 * 1.0 - 1.5 * 1.01 \times 10^5 * 0.5) = 11.36 \times 10^4$ J

   (3) $\Delta U = \frac{3}{2}(1.0 * 1.01 \times 10^5 * 1.0 - 1.5 * 1.01 \times 10^5 * 1.0) = -7.64 \times 10^4$ J

   (4) $\Delta U = \frac{3}{2}(1.0 * 1.01 \times 10^5 * 0.5 - 1.0 * 1.01 \times 10^5 * 1.0) = -7.64 \times 10^4$ J

   (d) $\Delta U_{total} = 0$ (take care of rounding errors!)

   (e) $\Delta W = -\int P dV$

   (1) $dV = 0 \quad \rightarrow \quad \Delta W = 0$

   (2) P = cont, so $\Delta W = -P\Delta V = -1.5 * 1.01 \times 10^5 * (1.0 - 0.5) = -7.57 \cdot 10^4$ J

   (3) $dV = 0 \quad \rightarrow \quad \Delta W = 0$

   (4) $\Delta W = -P\Delta V = -1.0 * 1.01 \times 10^5 * (0.5 - 1.0) = 5.05 \cdot 10^4$ J

   (f) So ΔW *by gas* $= -7.57 \cdot 10^4 + 5.05 \cdot 10^4 = 2.52 \cdot 10^4$ J

   (g) In each stage heat enters or leaves the system (as well as the work done). During part (1) and (2) heat must have been put into the system.

   (h) The work done is the area bounded by the path in the PV diagram.

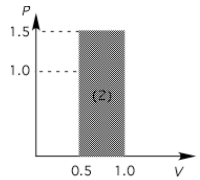

Figure 2: Work done by gas in (2)

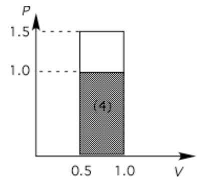

Figure 3: Work done on gas in (4)

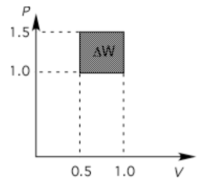

Figure 4: Total work done by gas

4. (a) $N = \dfrac{PV}{k_BT} = \dfrac{1.01 \cdot 10^5 \times 1}{1.38 \times 10^{-23} \cdot 300} = 2.42 \times 10^{25}$ molecules.

   $N_m = N/N_A = 2.42 \times 10^{25}/6.02 \times 10^{23} = 40.1$ moles

   $m = N_m * A$ (atomic mass) g = 40.1 * 40.0 = 1.6 kg

   (b) $C_V = \frac{3}{2} Nk_B = \frac{3}{2} N_m R = \frac{3}{2} * 40.1 * 8.31 = 500 \text{ JK}^{-1}$

   $C_P = \frac{5}{2} * 40.1 * 8.31 = 833 \text{ JK}^{-1}$

   $C_P/C_V = 1.67$.

   (c) $c_V = C_V/m = 500/1.6 = 312 \text{ JK}^{-1}\text{kg}^{-1}$

   $c_P = C_V/m = 833/1.6 = 521 \text{ JK}^{-1}\text{kg}^{-1}$

   $c_P/c_V = 1.67$.

   (d) $C_{V_m} = C_V/N_m = 500/40.1 = 12.5 \text{ JK}^{-1}\text{mol}^{-1}$

   $c_{P_m} = C_V/N_m = 833/40.1 = 20.8 \text{ JK}^{-1}\text{mol}^{-1}$

   $C_{P_m}/C_{V_m} = 1.67$.

5. (a) Blocks end up at same temp T and have same change in heat, so
$C(T-323)*5+C(T-273)*10=0 \quad\rightarrow\quad 15T=(273*10)+(323*5)$
$\rightarrow T=289.7\text{K}=16.7°\text{C}$

(b) Mass exhaled each breath $=0.5*1.3\times10^{-3}=6.5\times10^{-4}$ kg
Energy lost per breath $=1020*6.5\times10^{-4}*(37-20)=11.3$ J
Total per hour $=11.3*20*60=13.6$ kJ (equivalent to 3.8 W)

(c) Heat flow due to radiation $=\sigma(T_1^4-T_2^4)*A=5.67\times10^{-8}(308^4-293^4)*5\times10^{-2}=$ 4.6 W
So heat loss in one hour = 4.6 * 3600 = 16.6 kJ

(d) Total energy change = 50 * 100 * 360 = 1.8×10^7 J
So $\Delta T=\Delta Q/C=\dfrac{1.8\times10^7}{1020*1.3\times10^{-3}*4\times10^6}=3.4°\text{C}$

6. (a) $\text{d}Q=0$ so first law becomes $\text{d}U=-P\text{d}V=-\dfrac{Nk_BT}{V}\text{d}V$ (using ideal gas EOS)
also $U=\dfrac{n_d}{2}Nk_BT\rightarrow \text{d}U=\dfrac{n_d}{2}Nk_B\text{d}T$
equate the above for $\text{d}U\rightarrow -\dfrac{Nk_BT}{V}\text{d}V=\dfrac{n_d}{2}Nk_B\text{d}T$
divide through to get: $\dfrac{\text{d}T}{T}=-\dfrac{2}{n_d}\dfrac{\text{d}V}{V}$.

(b) $\ln T=-\dfrac{2}{n_d}\ln V+c_1=\ln V^{-2/n_d}+\ln c_2$ (c_1 and c_2 are constants)
$\rightarrow \quad TV^{2/n_d}=c_2$.

(c) Using $c_3*T=PV \quad\rightarrow\quad P*V*V^{2/n_d}=c_2/c_3=c_4$ (c_3 and c_4 are constants)
$\rightarrow \quad PV^{(2+n_d)/n_d}=c_4$
but $\gamma=\dfrac{C_P}{C_V}=\dfrac{(\frac{n_d}{2}Nk_B+Nk_B)}{\frac{n_d}{2}Nk_B}=\dfrac{n_d+2}{n_d}$
So $PV^\gamma=c_4$ For ideal gas $n_d=3$, so $\gamma=5/3$

(d) $P_1V_1^\gamma=P_0V_0^\gamma\rightarrow P_1=P_0(V_0/2V_0)^\gamma=P_0(1/2)^{5/3}\approx0.315P_0$
For isotherm, $P_1V_1=P_0V_0\rightarrow P_1=P_0(V_0/2V_0)=0.5P_0$
So adiabat is steeper than the isotherm.

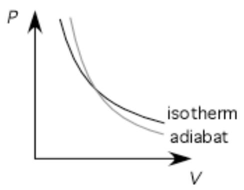

Figure 5: Isotherm and adiabat

7. (a) $\text{d}Q=0\rightarrow \text{d}W=\text{d}U=C_VdT$
$\rightarrow \Delta W=\int W=\int U=C_V(T_1-T_0)$

(b) $\Delta W=-\int P\text{d}V=-\int AV^{-\gamma}\text{d}V=-\dfrac{A}{1-\gamma}\left[V^{1-\gamma}\right]_{V_0}^{V_1}$
where A is a constant, but $AV^{-\gamma}=P$, so
$\Delta W=\dfrac{1}{(\gamma-1)}\left[P_iV_i\right]_0^1=\dfrac{1}{(\gamma-1)}(P_1V_1-P_0V_0)$

(c) $\gamma-1=(n_d+2)/n_d-1=2/n_d$, so
$\Delta W=\dfrac{1}{(\gamma-1)}(P_1V_1-P_0V_0)=\dfrac{Nk_B}{2/n_d}(T_1-T_0)=(n_dNk_B/2)(T_1-T_0)$
$\rightarrow \Delta W=C_V(T_1-T_0)$ as before

8. (a) $v_s=\sqrt{\dfrac{\gamma P}{\rho}} \quad\rightarrow\quad \gamma=\dfrac{n_d+2}{n_d}=\dfrac{\rho v_s^2}{P}=\dfrac{nmv_s^2}{nk_BT}$
$n_d+2=n_d\dfrac{mv_s^2}{k_BT} \quad\rightarrow\quad n_d\left(\dfrac{mv_s^2}{k_BT}-1\right)=2$
$\rightarrow \quad n_d=2\left(\dfrac{mv_s^2}{k_BT}-1\right)^{-1}$

(b) $n_d=2\left(\dfrac{4.82\times10^{-26}\cdot344^2}{1.38\cdot10^{-23}293}-1\right)^{-1}\approx5$

(c) air is mostly diatomic, so might expect $n_d=7$, but reason is explained in lec. 7