Vibrations & Waves
Problem Sheet 1: QUESTIONS
Covers material in V & W Lectures 1 & 2

1) Upon solving the Equations of Motion, some simple harmonic oscillators were found to follow:

(i) $x(t) = 2\exp(\,i6t)$

(ii) $x(t) = i\,3\exp(\,i5t)$

(iii) $x(t) = (2 + i\,3)\exp(\,i6t)$

(iv) $x(t) = (1 - i5)\exp(\,i2t)$

Find the *real part* of these solutions.

2) Some oscillators were observed to obey the following equations:

(i) $x(t) = 5\cos(8t)$

(ii) $x(t) = 5\cos(8t + 0.2\pi)$

(iii) $x(t) = 7\cos(5t - 0.3\pi)$

(iv) $x(t) = 5\sin(7t)$

Rewrite these equations in the *complex notation* of the form:

$x(t) = (a + ib)\exp(\,i\omega t)$

3) A spring is hung vertically from a support. A mass is attached to the end of the spring. The vertical displacement of the mass is observed to follow the equation:

$$x(t) = 0.05\cos(7.51t)$$

where everything is in SI units. What, in SI units, is (i) the amplitude A, (ii) the angular frequency ω, (iii) the frequency f and (iv) the period T. If the mass is 0.1 kg, what is the spring constant s of the spring?
How far would the spring have stretched when the mass was initially attached to its end?
Take $g = 9.8\ \mathrm{ms^{-2}}$.

4) The motion of the horizontal mass on a spring has the general solution:

$x(t) = A\cos(4t + \varphi)$

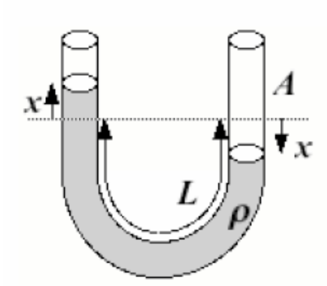
Work out the variation of the velocity v(t) with time. By considering the initial conditions, work out the value of A and φ for the following cases:

(i) t = 0, x = 0.3 m, v = 0

(ii) t = 0, x = -0.5 m, v = 0

(iii) t = 0, x = 0, v = 1.2m/s

5) A U shaped tube of cross-sectional area A is filled with a liquid of density ρ. A total length L is filled, as illustrated below:



The tube is tilted so that the liquid is displaced by +h on one side and –h on the other side. It is then returned instantaneously to the vertical.

(i) Show that the Equation of Motion is given by:

$$LA\rho \frac{d^2x}{dt^2} = -2A\rho g x$$

(ii) By solving the Equation of Motion, find the variation of the vertical displacement x(t) of the liquid with time t.
(iii) At what angular frequency ω_0 does the liquid oscillate?
(iv) What is the velocity v(t) of the oscillating liquid?
(v) What is the acceleration a(t)?
(vi) Derive the variation of the potential energy PE of the liquid with x and t.
(vii) Derive the variation of the kinetic energy KE of the oscillating liquid with t.
(viii) Find the total energy TE of the oscillating liquid.
(ix) Find the variation of KE with x.

6)* (more challenging) Show that the restoring force about any stable equilibrium point is linear (i.e. Hooke's Law is valid) for sufficiently small displacements. Since a linear restoring force leads to SHM, this explains why SHM is so widespread and important. [Hint: Consider the potential U(x) which is related to the force by F(x) = -dU(x)/dx. Make a Taylor series of U(x) about the equilibrium point x_0 and then think what is meant by x_0 being a stable equilibrium point.]