Vibrations & WavesClasswork 4: Questions

a) Which of the following waves (written in vector form) are transverse waves and which are longitudinal ($\underline{\hat{\mathbf{x}}}$, $\underline{\hat{\mathbf{y}}}$ and $\underline{\hat{\mathbf{z}}}$ are unit vectors in the x,y and z directions)? Which are travelling waves and which are standing waves?

i) $\underline{\boldsymbol{\psi}}(z,t) = A\underline{\hat{\mathbf{x}}}\cos(\omega t - kz)$

ii) $\underline{\boldsymbol{\psi}}(z,t) = A\underline{\hat{\mathbf{z}}}\cos(\omega t)\sin(kz)$

iii) $\underline{\boldsymbol{\psi}}(x,t) = A\underline{\hat{\mathbf{x}}}\cos(\omega t - kx)$

iv) $\underline{\boldsymbol{\psi}}(z,t) = A_1\underline{\hat{\mathbf{x}}}\cos(\omega t - kz) + A_2\underline{\hat{\mathbf{z}}}\cos(\omega t - kz)$

b) Consider two stretched strings. String 1 is twice as long and has half the total mass of String 2. The tensions in both strings are the same. The phase velocity of wave on String 1 is measured to be 5m/s. What is the phase velocity for waves on String 2?

c) The waves in a deep pond have a phase velocity given by $v_p = \sqrt{g/k}$ where g =9.8ms^{-2} is the acceleration due to gravity.

i) Show that this is the same as $v_p = a\sqrt{\lambda}$ (Eq. [7.9] in the lecture notes) with $a = \sqrt{g/2\pi}$.

ii) Hence show that $\omega = \sqrt{g}\sqrt{k}$ and check that this is dimensionally correct. Sketch ω versus k. What is the physical significance of (1) the ratio ω/k at a given k value and (2) the slope of the curve at a given k value? Recall that this kind of plot is known as the "dispersion curve" for the medium.

A large stone is dropped into a 10m-wide pond at one side. This creates a range of cosine-type travelling waves of wavelengths from 0.1m to to 0.4m which add together (superpose) to form a wavegroup (or pulse).

iii) What is the group velocity of the pulse? Hint: calculate v_g at the mean wavelength.

iv) How long does it take the pulse to cross the pond?

v) Compare your answer in iv) to the time taken for an isolated wave at the mean wavelength of the group to cross the pond.

d) This question is to test your *general* understanding of coupled oscillations. So you should be able to do most of it without consulting your lecture notes too much.

The general solution for the displacements of two identical simple pendulums A and B coupled by a spring is

$$x_A(t) = \frac{A_1}{2}\cos(\omega_p t + \varphi_1) + \frac{A_2}{2}\cos(\omega_c t + \varphi_2)$$

$$x_B(t) = \frac{A_1}{2}\cos(\omega_p t + \varphi_1) - \frac{A_2}{2}\cos(\omega_c t + \varphi_2)$$

where A_1, A_2, φ_1 and φ_2 are constants, ω_p is the frequency of a single pendulum of length L, $\omega_c = \sqrt{\omega_p^2 + 2\omega_s^2}$ is the coupled frequency and ω_s is the frequency for a mass m on a spring of spring constant s.

i) What are the velocities $v_A(t)$ and $v_B(t)$?

ii) Use simple sketches to show the motion of the pendulums corresponding to the two normal modes of the system. From these write down two sets of expressions for x_A and x_B corresponding to these normal modes.

iii) Write down as simply as possible two sets of initial conditions for x_A, x_B, v_A and v_B which you would expect to excite each normal mode independently. Taking $\varphi_1 = 0$ (to make the maths a bit clearer), check your answers by applying your initial conditions to the general solution.

iv) If $m = 0.1$kg, $L = 1$m and $s = 1.6$Nm^{-1} calculate the frequencies of the normal modes. Take g = 9.8ms^{-2}. Assuming a superposition of normal modes whose frequencies "beat", what is the time interval between moments of maximum displacement of one of the pendulums.