Professional Skills for Physicists: II

Problem Solving: Section C

A physics toolkit, energy, diagrams and idealised models

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12 A physics toolkit

In the course of learning physics you build up a toolkit of concepts and equations. When faced with the questions asked at the end of section B, the crucial first step is to select the right tools from the toolkit. To evaluate the importance of gravity in the atom, the important tools were (i) the equation for gravitational potential (ii) the approximate relationship between density, the mass of a proton and the size of an atom (iii) the idea that kT is of order the thermal energy of an electron at temperature T (iv) that an electron cannot be bound to an atom by gravity if its kinetic energy is much greater than the gravitational potential energy.

To be a good physicist you need to build up your toolkit, and to develop the skill of choosing the right tool from it.

So, what are the basic results in physics that you carry around in your instant-recall memory all the time? Newton's laws of motion? the universal law of gravitation? Coulomb's law? the ideal gas laws? Ohm's law? ...the laws or equations of anyone else? E=mc^2? What about commonly used expressions that don't quite make it to the status of being a named law?

> **Exercise 12.1:** Review which of these concepts, or others, are at your fingertips, and compare with your colleagues. There is likely to be a lot in common and it can be instructive to consider what is 'most fundamental', and hence essential.

13 Energy is the most useful tool

The examples in the section 11 were mainly about energy: energy is the capacity to do work, so the physical effect with the most energy associated with it will have the greatest capacity to do work and therefore dominate other effects. Hence, the associated energy is a good guide to the relative importance of contributing physical effects. For example, the gravitational energy in an atom was shown in section 10 to be so small that gravity cannot play a significant role in holding atoms together.

> **Exercise 13.1:** Think of another case in which an energy argument can be used to show that a physical effect is unimportant.

14 An energy toolkit

Many examples in the previous sections compared one form of energy with another, so an important part of the physicist's toolkit is a list of different expressions for energy. Some of the most common are:

Kinetic energy: $\frac{1}{2}mv^2$

Energy in a (uniform) gravitational field: mgh

Gravitational potential energy: Gm_1m_2/r

Electric potential energy: $q_1q_2/4\pi\epsilon_0 r$

Electric field energy density: $\epsilon_0 E^2/2$

Magnetic field energy density: $B^2/2\mu_0$

Energy density of perfect monatomic gas, where n is number density: $3nkT/2 = 3P/2$

Thermal energy take-up in a substance of specific heat capacity c: $mc\Delta T$

Kinetic energy density of fluid: $\rho v^2/2$

Photon energy: $h\nu$

Rotational energy: $I\omega^2/2$

Energy in a capacitor: $Q^2/2C = CV^2/2$

Energy in an inductance: $LI^2/2$

Further reasons why energy is so useful are that (i) it is a global quantity referring to an entire system, not just one part (ii) energy is always conserved. Applying the principle of the conservation of energy is the only way to track energy as it is transferred from one form to another, or split (e.g. between bulk motion and thermal internal energy, as in friction).

15 Another toolkit item: physical flux

This is a concept, of considerable utility, that is often treated as 'common sense' and hence not given much emphasis in formal discussions of e.g.

electromagnetism, structure of matter, quantum mechanics and so on. It is included here as a handy toolkit item.

It is useful to pair consideration of physical flux with the concept of density as follows:

Density: You already know that a density is a measure of the amount of something per unit volume. The 'something' can be: number of particles; mass of a substance; or energy. Conventionally these different densities would be represented by the symbols: n, ρ, U (or sometimes u).

Flux: In Physics, a flux is understood to be a measure of the rate at which 'something' (gas atoms/molecules, photons ...a fluid) passes through unit area of a surface per unit time. Consider atoms in a monatomic gas passing through a hole in the wall of its container. How many leave per unit time? If the hole is of unit area, then the quantity we are seeking is the flux out of the container. Take a time interval Δt, and suppose that we know that the *mean* value for the component of the atoms' velocity directed toward and perpendicular to the plane of the hole is v_x. If the number density of atoms is n, then the number passing through the hole in Δt is:

$$nv_x\Delta t$$

You can think of $v_x\Delta t$ as the furthest distance an atom can be from the hole, at the start of the timing period, if it is to reach it within time Δt.

Since the flux ϕ is the number passing through this unit area per unit time, the above expression simply has to be divided by Δt to give:

$$\phi = nv_x$$

To change this flux into a mass or an energy flux, you would just have to multiply the rhs by m, the mass of each atom, or by the energy E ($= 3kT/2$) carried, on average, by each atom.

Hence, notice that a flux is the product of a density and a velocity, fundamentally - if you know any two of the quantities, you know the third.

Vector treatment: What if the flow under consideration is not directed at right angles with respect to the surface through which it passes? In this more general case, the flux would involve the scalar product between the flow velocity vector and the normal to the surface (e.g. $\rho\mathbf{v}.\hat{\mathbf{n}}$, where $\hat{\mathbf{n}}$ is the unit vector normal to the surface).

Finally, a flux out of a surface will often be linked to the amount of stuff added to the flow inside the surface. In the case of sunlight, originating from a conveniently (very nearly) spherical object, it is easy to visualise. If L is the Sun's radiant luminosity (total light energy produced per unit time), its flux ϕ at a distance D will be that energy rate spread over a sphere of surface area $4\pi D^2$: that is, $\phi = L/4\pi D^2$.

16 Toolkit estimations and problems

Exercise 16.1: In each of the following, an order of magnitude estimate is required. Estimate

(i) the gravitational energy of a 100kg satellite in low earth orbit, the velocity of the satellite, and the period of the orbit.

(ii) the kinetic energy of a car travelling at 70mph and the power output of a car engine.

(iii) the mass of the brakes of a train needed to stop the brake block temperature reaching melting point when the brakes are applied

(iv) the number of photons emitted per second by a light bulb

(v) the magnitude of the oscillating optical-frequency electric and magnetic fields in the vicinity of a light bulb.

(vi) the mass of a helium nucleus is about 1 percent less than the mass of four hydrogen nuclei. Given this and that the Sun's hydrogen-burning lifetime is believed to be 10 Gyr, derive an estimate for the Sun's present-day luminosity.

(vii) estimate the power of the sunlight per unit area when it reaches the Earth. (Notice this is the same as the flux of sunlight.)

Exercise 16.2:

(i) Estimate the radius of a solar mass (2×10^{30} kg) black hole.

(ii) Given that a black hole emits black body radiation at a temperature (in degrees Kelvin) given by $\hbar c^3/(8\pi MGk)$, how long would it take a solar mass black hole to evaporate? If small black holes were created during the big bang, what mass of black hole would now be evaporating?

Exercise 16.3: At the end of popular TV programmes, the boiling of kettles creates a significant surge in the demand for electricity. Estimate the power output of the national grid.

Exercise 16.4: Using the result from exercise 16.1(vii), estimate the area of land needed to be covered by solar cells if solar energy were to generate the nation's electricity.

Exercise 16.5: A radio station broadcasts a 105 kW signal at 97.9 MHz. Estimate the following quantities:

(i) the wavelength of the emitted radiation,

(ii) the photon energy of the emitted radiation,

(iii) the number of photons emitted per second,

(iv) the photon flux at a distance of 1.5 km from the radio mast,

(v) the energy density of the emitted radiation at 3 km from the radio mast,

(vi) the maximum distance at which a signal can be detected by a portable radio that has a sensitivity of 3 μW.

Exercise 16.6: Estimate the maximum power that could be generated from the wind if the entire surface area of the UK were covered with windmills.

Exercise 16.7: Estimate the energy needed to inflate a car tyre.

It is useful to remember that a force applied through a distance is energy: energy$= \int F ds$. This relationship lies at the base of the approximate equivalence of pressure and energy density. Equivalently, power is a force times a velocity, and force is an energy potential divided by (or differentiated with respect to) distance.

Exercise 16.8: Estimate the maximum speed of a car as limited by wind resistance (the result obtained in exercise 16.1(ii) may be useful).

17 Drawing Diagrams

Sometimes when faced with a problem, it is very difficult to know where to start. Often the most difficult part of problem-solving is turning the words of the question into a picture of what is going on. Sometimes, making yourself sketch a diagram describing the situation helps you break through. In the following exercises, begin by drawing a diagram.

Exercise 17.1: Playing snooker, your white ball is on the brown spot and a red ball is at the mid-point between the two pockets at the middle of the table. How should the white ball hit the red ball to propel it towards one of the pockets at the bottom of the table. You can assume that a snooker table is twice as long as it is wide. If the ball is frictionless, by what distance (as a fraction of the ball radius) should the centre of the white ball be aimed to miss the centre of the red ball?

Exercise 17.2:

A man standing on the south bank of a still trough of water of width w, which lies east-west, can run with speed V_1 and swim with speed V_2 ($< V_1$). What path should he take to reach a point on the north bank of the water trough, a distance d to the east, in the shortest possible time?

Exercise 17.3: A lunar spacecraft sits in a low circular orbit at a height of 200km above the Moon's surface. Suppose the spacecraft is divided into two parts with equal mass and the two parts separate with a relative velocity, v_{split} along the direction of motion (take v_{split} to be smaller in magnitude than the original orbital speed). Sketch plots of the subsequent trajectories of each part. If one part were to just graze the surface of the moon, where would it do so relative to the point of separation? Review, qualitatively, what will happen in the case of separation perpendicular to the original motion.

Exercise 17.4: In exercise 8.3 you showed that the sound speed in a gas is $c_s = \text{constant} \times \sqrt{kT/m}$. Assuming that the constant in this expression is equal to one, derive approximately the minimum ceiling height of a church needed to house an organ with a pipe that sounds four octaves below middle C.

Exercise 17.5: At night, a commuter looks through an umbrella (ie through the material) at a distant streetlight and sees a regular pattern of bright and dark fringes around the streetlight. The distance between the apparent bright fringes is described as being about 1 mm projected onto the umbrella held at arm's length. Quantitatively, is it reasonable to accept that this fringe pattern is a diffraction pattern?

Exercise 17.6: In a World War II sea battle in the North Atlantic, two opposing battleships are due north and south of each other separated by a distance of 10km. With their guns pointed directly south and north respectively, they launch shells at each other. The paths of the shells deviate from a north-south line due to the Earth's rotation. In which direction do they deviate, and by how much? Do the gunners need to take account of this?

Exercise 17.7:

Via spectroscopic observation, astronomers are able to measure the component of a star's space velocity projected onto the line of sight (the so-called radial velocity of the star).

Consider the case of a bright star in a circular orbit within a binary system, where the second star is a dark object. The bright star's orbit speed is 120 km s^{-1}, and the period of orbit is 0.8 days. The binary system is 48 light years distant, and its centre of mass moves towards the Earth at a speed that is much smaller than the bright star's orbital speed.

(a) In the case that the plane of the bright star's circular orbit is viewed edge-on by observers on Earth, sketch and label a graph to show how the star's observed radial velocity would vary as a function of time. (Adopt as your convention that positive velocity implies motion towards the Earth.)

(b) Now suppose that the invariance of the speed of light claimed in Einstein's Special Theory of Relativity is wrong! Instead, imagine that the speed of light obeys Galilean transformations such that $c \longrightarrow c + v$ when emitted by an object travelling at speed v with respect to the observer. In such circumstances, sketch how the radial velocity curve for the bright star in (a) would then appear to an Earth-bound observer, specifying relevant timescales. (Consider only the star's apparent line-of-sight motion, and ignore any motion by the Earth itself).

18 Idealised models

Consider how you might attempt the following exercise before reading on:

Exercise 18.1:

Use the fact that the temperature of the Earth's atmosphere is approximately 300 K to estimate its depth.

This exercise does not have an exact answer since the atmosphere tails off exponentially into a near-vacuum. Other difficulties of the question are that the answer depends on the variation with height of a number of parameters, such as temperature, composition and degree of ionisation. Nevertheless, it is a meaningful question in the sense that there is some height over which the density of the atmosphere drops significantly, say by 50 percent, and we can certainly make an order of magnitude estimation. The way to overcome the difficulties of the question is to set up an idealised model of the atmosphere and calculate its height. A suitable ideal model might be a simple visualisation of the atmosphere as a slab of gas with height h. Within the slab, we assume that the molecular number density (n), temperature (T), mass (m) of each molecule are constant, and that the gas is not ionised or dissociated into atoms. In this idealised model, the weight of the atmosphere above unit area of the Earth is $nmgh$ and the pressure at ground level is nkT. The pressure supports the atmosphere, so $nkT = nmgh$, giving $h = kT/mg$. Substituting $T = 300$K, $m = 30m_p$ (a mixture of O_2 and N_2), and $g = 10$ms^{-2}, gives a height h of around 100km.

The construction of an idealised model is frequently needed in problem-solving. This is a skill you will already have used to solve many of the problems set in previous sections. The aim of this section is just to make you more conscious of the use of idealised models.

Exercise 18.2:

Consider an isolated system of mass M taking up a volume V, with the particles making up the mass interacting only through gravity. (This shall be turned into a naive model of the Sun.)

(a) Using dimensional analysis, construct the timescale for the evolution of this system. That is, how can we combine G, the universal gravitation constant, with M and V to make a quantity with the units of time?

(b) Make a rough numerical estimate of the timescale of gravitational evolution of the Sun in the absence of other forces.

(c) This can be looked at another way - suppose the internal pressure gradient supporting the Sun against gravity was suddenly removed. Describe what would then happen and use simple physical arguments to derive an approximate expression for the timescale associated with the change.

Exercise 18.3:

Some geologists predict that a large part of the island of La Palma (one of the Canary Islands, area approx. 750 km^2) will break off during the next volcanic eruption on the island and trigger a huge tsunami.

(a) When the water depths are not too shallow, most of the energy that is stored in a tsunami is gravitational potential energy due to the displacement of the water masses. It has been argued that e, the energy per unit length of the wave crest, is given by

$$e = \alpha\rho(hd)^{3/2}$$

where α is a dimensionless constant of the order of 0.1, ρ is the mass density of water, h is the wave height and d is the water depth. Check whether this equation is dimensionally plausible, and, if not, suggest a modification.

(b) Approximate the consequences of the massive eruption by assuming the whole of La Palma slumps 100 m into the Atlantic. Derive an expressions that shows how e would change with distance as the resulting tsunami wave spreads away from the island? Estimate a typical tsunami wave height in (i) the mid-Atlantic, and (ii) close to the US east coast, 5000 km away. (For simplicity, you can assume that the bulk of the energy is indeed potential energy and is stored in just a single wavefront. You can also assume that the wave travels in water deep enough so that it does not lose energy via its interaction with the sea floor).

Exercise 18.4: (i) A star similar to the sun is observed to explode. Estimate the critical explosion velocity that must be exceeded for the star to continue expanding and avoid recollapse due to gravity.

(ii) Estimate the critical density of the present-day visible universe required to make the universe collapse and stop it expanding for ever.

(iii) Given that galaxies contain typically 10^{11} stars, and that the typical distance between galaxies is 1 Mparsec, investigate whether there is sufficient visible matter in the universe to make it open (expand forever) or closed (recollapse).

(iv) Consider two galaxies separated by a large distance. How must this distance vary with time if the universe is to be forever on the boundary between being closed and open.

The doubt about idealised models is whether they really are equivalent to the reality they represent. In the above exercise, the neglect of General Relativity (GR) places a big question-mark over the simple treatment, but as it happens it does give reasonable answers.

Idealised models are essential starting points in research. Faced with a phenomenon you do not understand, it is useful to try out idealised hypothetical models first and see whether the answer begins to fit the data. A simple model achieving a good fit is always going to be worth more, in the long run, than a more sophisticated one (usually demanding more inputs) that does not do significantly better. Simplicity is not to be despised! Exactly this simplicity or 'parsimony' is enshrined in the concept of Occam's Razor. Ultimately, we rule in favour of GR over the simple explosive model for the Universe using Occam's Razor, because GR works across so many more problems.