First Year **Relativity** **Problem Sheet 2** 16Dec04

1. In problem 2, in the last problem sheet, you were asked to calculate the speed necessary to reach Alpha Centauri (distance 4.4 light years from Earth) in 10 years as measured by the rocket clocks. Now extend the analysis to the twin problem. Presume that on reaching Alpha Centauri the travelling twin immediately turns around and returns home at the same speed. How much younger will the travelling twin be on his return to earth with respect to his brother who remained at home? Why is it incorrect for the travelling twin to conclude that from his perspective the earth has been receding, and then approaching, so that it is he, not his brother, who ages faster? (*Hint: Think carefully about what each brother feels between departure and meeting up again – does each brother have an identical history?).*

2. An observer in the lab frame sees two particles travelling respectively at $V = +0.8c$ and $V = -0.8c$ along the x - axis. (a) With what velocity does an observer stationary with respect to the first particle see the second particle travel? (b) Similarly with what velocity does the first particle move with respect to the second? (c) Finally, what is the relative velocity between the two particles as observed from the laboratory (*care*!)? Comment briefly on your answer to part (c).

3. An observer O' moves along the positive x - axis with velocity $u = 0.9c$ relative to an observer O. The space-time origins of O and O' coincide. Use Lorentz transformations to answer the following:
   (a) What are the space-time coordinates in O' of events *A* and *B* occurring with coordinates $(x_A, t_A) = (0, 100\,\text{ns})$; $(x_B, t_B) = (-100m, 0)$ in O ?
   (b) What are the space-time coordinates in O of events *C* and *D* occurring with coordinates $(x'_C, t'_C) = (0, 100\,\text{ns})$; $(x'_D, t'_D) = (-100m, 0)$ in O' ?
   (c) What are the momentum and energy in O' of an electron ($m = 0.5\,\text{MeV/c}^2$) with momentum $p = 0.5\,\text{MeV/c}$ along the +ve x - direction in O?
   (d) What are the momentum and energy in O of an electron ($m = 0.5\,\text{MeV/c}^2$) with kinetic energy, $K = 1\,\text{MeV}$ in O' ?

   Answers: (a) (-62m,230ns), (-230m, 700ns), (b) (62m,230ns), (-230m,-700ns)
   (c) (-0.32MeV/c, 0.60MeV) , (d) (6.35MeV/c, 6.37MeV).

4. Express the following in S.I. units ($1eV = 1.6\times10^{-19} J$)
   (a) The mass of an electron, $m_e = 0.511 MeV / c^2$
   (b) The rest energy of an electron.
   (c) The energy liberated when an electron and a positron annihilate at rest.
   (d) The energy of cosmic-rays of the highest observed energy ($\sim 10^{19} eV$).

   Answers: (a) $0.9\times10^{-30} kg$, (b) 0.08pJ , (c) 0.16 pJ , (d) 1.6 J.

5. *Time dilation in particle physics*: The neutral pion has a mass $m_\pi = 135 MeV / c^2$ and its mean-life (in its rest frame) is $\tau_\pi = 10^{-16}$ sec.
   Calculate the mean flight path (in metres) before decay of:
   (a) A pion with velocity $0.1c$
   (b) A pion with momentum $1 GeV / c$.
   (c) A pion with kinetic energy $1 TeV$.

   Answers: (a) 3nm, (b) 220 nm, (c) 220 μm

6. *Relativistic Doppler Effect:* Following a relativity lecture, with her brain occupied by new ideas in physics, a first year student drives along the road in her car and goes through a red light. At the court case the student, in her defence, tells the judge that in driving towards the light it appeared green due to the Doppler effect. The judge accepted this plea and changed the charge to one of speeding. Taking values of wavelengths

($\lambda_{red} = 650nm$; $\lambda_{green} = 530nm$), use the Doppler formula $\frac{\lambda}{\lambda'} = \left(\frac{1+u/c}{1-u/c}\right)^{1/2}$ where the source (at rest in O) *approaches* the observer O', to estimate the fine imposed if the standard rate is £10 for each mile per hour in excess of the limit (30 mph).

7. *Fission and Fusion*: (a) If the fission of one nucleus of ${}^{235}_{92}U$ releases $200MeV$, estimate the consumption in kg per year of this fuel in a fission reactor with a power output of 1GW. Assume the efficiency is 33% and take a value $N_A = 6\times10^{26}(kg\ mole)^{-1}$. (b) One D-D (D is the symbol for deuterium, the heavy isotope of hydrogen ${}^{2}H$) fusion reaction releases 4MeV. The mass ratio of deuterium to hydrogen in sea water is 1/3250. What is the fusion energy available in sea water (Joules per kg)?